

الف) $y^3 = 2x^2 - 1$

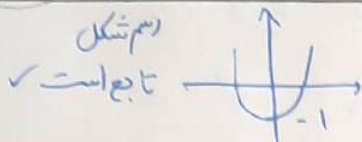
if $(x_1, y_1) \in f$
if $(x_2, y_2) \in f$ $\left\{ \begin{array}{l} x_1 = x_2 \\ y_1 = y_2 \end{array} \right.$

$\left\{ \begin{array}{l} y_1^3 = 2x_1^2 - 1 \\ y_2^3 = 2x_2^2 - 1 \end{array} \right. \xrightarrow{x_1 = x_2} y_1^3 = y_2^3 \rightarrow y_1 = y_2$
تابع است ✓

ب) $y^3 + y = 3x^2 - 1$

$\left\{ \begin{array}{l} y_1^3 + y_1 = 3x_1^2 - 1 \\ y_2^3 + y_2 = 3x_2^2 - 1 \end{array} \right. \xrightarrow{x_1 = x_2} y_1^3 + y_1 = y_2^3 + y_2$
 $\rightarrow y_1^3 - y_2^3 + y_1 - y_2 = 0$
 $(y_1 - y_2)(y_1^2 + y_1 y_2 + y_2^2) + (y_1 - y_2) = 0$
 $(y_1 - y_2)(y_1^2 + y_1 y_2 + y_2^2 + 1) = 0$
 $\rightarrow y_1 - y_2 = 0 \rightarrow y_1 = y_2$ تابع است ✓

الف) $y = x^2 - 1$



ب) $|y| + xy = x^2 + 1$

مثال نقیض

$x=0 \rightarrow |y|=1 \rightarrow y=\pm 1$ تابع نیست ✗

ج) $y^3 + x^3 = 2x - 1$

صرف ریاضی تابع است ✓

$\left\{ \begin{array}{l} y_1^3 + x_1^3 = 2x_1 - 1 \\ y_2^3 + x_2^3 = 2x_2 - 1 \end{array} \right. \xrightarrow{x_1 = x_2} y_1^3 = y_2^3 \rightarrow y_1 = y_2$

د) $\sqrt[3]{y} + \sqrt[3]{x} = 2$

$\sqrt[3]{y} = 2 - \sqrt[3]{x} \rightarrow y = (2 - \sqrt[3]{x})^3$ تابع است ✓

الف) $\tan \frac{\pi}{3} (60^\circ) \rightarrow \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{2\sqrt{3}}{2} = \sqrt{3}$

د) $\cot 180^\circ = \frac{-1}{0}$ تعریف نشده ✗

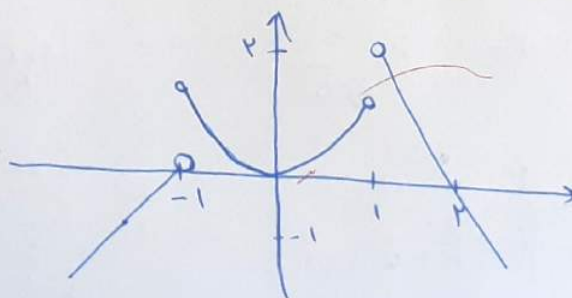
ب) $\cot \frac{\sqrt{\pi}}{4} (210^\circ) \rightarrow \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{-2\sqrt{3}}{2} = -\sqrt{3}$

ج) $\tan 330^\circ \rightarrow \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{-2}{2\sqrt{3}} = \frac{-1}{\sqrt{3}}$

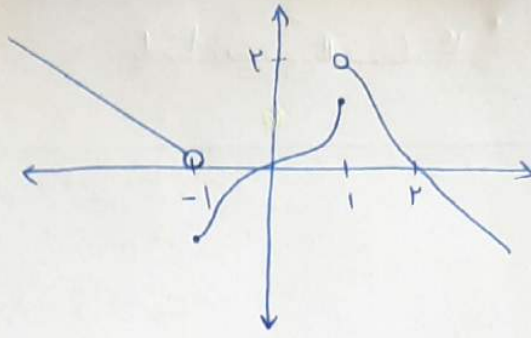
الف) $\sin \alpha = \frac{1}{2}$ $\left| \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} \right| \left| \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} \right| \rightarrow \alpha = \frac{\pi}{6}, \frac{5\pi}{6}$

$\rightarrow \cos \alpha = \frac{\sqrt{2}}{2}$ $\left| \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} \right| \left| \frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} \right| \rightarrow \alpha = \frac{\pi}{4}, \frac{3\pi}{4}$

$f(x) = \begin{cases} -x+4 & x > 1 \\ x^2 & -1 < x < 1 \\ x+1 & x < -1 \end{cases}$

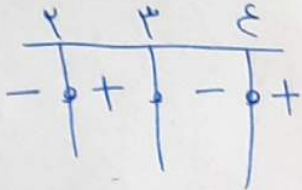


$$f(x) = \begin{cases} -x+3 & x > 1 \\ x^3 & -1 \leq x \leq 1 \\ -x-1 & x < -1 \end{cases}$$



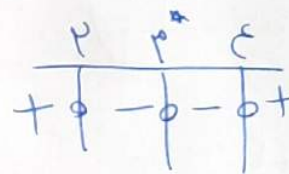
الف) $y = (x-2)(x-3)(x-4)$

2 3 4



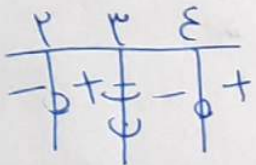
ب) $y = (x-2)(x-3)^2(x-4)$

2 3* 4



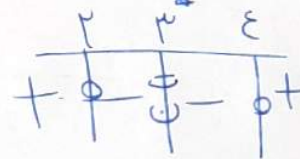
الف) $y = \frac{(x-2)(x-4)}{x-3}$

2 3 4



ب) $y = \frac{(x-2)(x-4)}{(x-3)^2}$

2 3* 4



الف) $y = \frac{x^3-3x+2}{x-3}$

2 3

Polynomial division:

$$\begin{array}{r} x^2 + x - 1 \\ x-3 \overline{) x^3 - 3x + 2} \\ \underline{x^3 - 3x^2 + 3x - 3} \\ 3x^2 - 4x + 5 \\ \underline{3x^2 - 9x + 9} \\ 5x - 4 \\ \underline{5x - 15} \\ 11 \end{array}$$

Remainder: 11

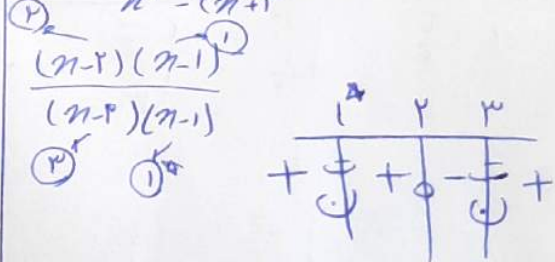
Factorization: $(x-3)(x^2+x-1) + 11$

Sign chart for $y = \frac{x^3-3x+2}{x-3}$:

2 3 4

ب) $y = \frac{x^3-3x+2}{x^2-4x+3}$

2 3 4



الف) $y = \frac{x^4-x^2}{x^2+x+2}$

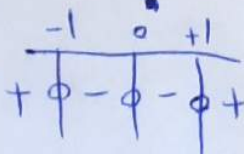
$\Delta = b^2 - 4ac = 1 - 4 \cdot 1 \cdot 2 = -7 < 0$

Discriminant is negative, so the denominator has no real roots and is always positive.

$x^4 - x^2 = 0$

$x^2(x^2 - 1) = 0$

$x = 0, \pm 1$



ب) $y = \frac{x^4+3x+4}{x^2+2x+3}$

$\Delta = b^2 - 4ac = 9 - 4 \cdot 1 \cdot 3 = -3 < 0$

Discriminant is negative, so the denominator has no real roots and is always positive.

$\Delta = b^2 - 4ac = 9 - 4 \cdot 1 \cdot 3 = -3 < 0$

Sign chart for $y = \frac{x^4+3x+4}{x^2+2x+3}$:

$x \in (-\infty, +\infty)$

Sign: +