

الف) $y^3 = x^2 - 1 \Rightarrow \begin{cases} y_1^3 = x_1^2 - 1 \\ y_2^3 = x_2^2 - 1 \end{cases} \xrightarrow{x_1 = x_2} y_1^3 = y_2^3 \Rightarrow$ تابع است.

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ب) $y^3 + y = x^2 - 1 \Rightarrow \begin{cases} y_1^3 + y_1 = x_1^2 - 1 \\ y_2^3 + y_2 = x_2^2 - 1 \end{cases} \xrightarrow{x_1 = x_2} y_1^3 + y_1 = y_2^3 + y_2 \Rightarrow$ تابع است.

چرا؟ باید راه حل کامل صد خنده نبوی:

الف) $y = x^2 - 1 \Rightarrow \begin{cases} y_1 = x_1^2 - 1 \\ y_2 = x_2^2 - 1 \end{cases} \xrightarrow{x_1 = x_2} y_1 = y_2$ تابع است. ج) $y^3 + x^3 = x^2 - 1 \Rightarrow \begin{cases} y_1^3 = -x_1^3 + x_1^2 - 1 \\ y_2^3 = -x_2^3 + x_2^2 - 1 \end{cases} \xrightarrow{x_1 = x_2} y_1^3 = y_2^3 \Rightarrow$ تابع است.

مثال نقیض

ب) $y^3 + y = x^2 - 1 \Rightarrow \begin{cases} y_1^3 + y_1 = x_1^2 - 1 \\ y_2^3 + y_2 = x_2^2 - 1 \end{cases} \xrightarrow{x_1 = x_2} y_1^3 + y_1 = y_2^3 + y_2 \Rightarrow$ تابع است. $y = 0 \Rightarrow 1y + 0 = 1 \Rightarrow 1y = 1 \Rightarrow y = \pm 1$ تابع نیست! $\sqrt[3]{y} + \sqrt[3]{x} = 2 \Rightarrow \begin{cases} \sqrt[3]{y_1} = -\sqrt[3]{x_1} + 2 \\ \sqrt[3]{y_2} = -\sqrt[3]{x_2} + 2 \end{cases} \xrightarrow{x_1 = x_2} \sqrt[3]{y_1} = \sqrt[3]{y_2} \Rightarrow y_1 = y_2$ تابع است. $y = 1 \Rightarrow 1y + 1 = x^2 - 1 \Rightarrow y_1 + 1 = x_1^2 - 1 \Rightarrow y_1 = x_1^2 - 2$ تابع است. $y = -1 \Rightarrow -1y + 1 = x^2 - 1 \Rightarrow -y_1 + 1 = x_1^2 - 1 \Rightarrow -y_1 = x_1^2 - 2 \Rightarrow y_1 = 2 - x_1^2$ تابع است. $y = 2 \Rightarrow 2y + 2 = x^2 - 1 \Rightarrow 2y_1 + 2 = x_1^2 - 1 \Rightarrow 2y_1 = x_1^2 - 3 \Rightarrow y_1 = \frac{x_1^2 - 3}{2}$ تابع است. $y = -2 \Rightarrow -2y + 2 = x^2 - 1 \Rightarrow -2y_1 + 2 = x_1^2 - 1 \Rightarrow -2y_1 = x_1^2 - 3 \Rightarrow y_1 = \frac{3 - x_1^2}{2}$ تابع است.

الف) $\tan\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{\frac{1}{\sqrt{2}}} = \sqrt{2}$

ب) $\cot\left(\frac{\sqrt{2}\pi}{4}\right) = \frac{\sqrt{2}}{\frac{1}{\sqrt{2}}} = \sqrt{2}$

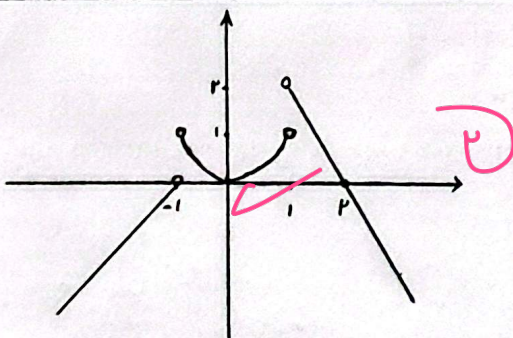
ج) $\tan(2\pi) = \frac{-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = -1$

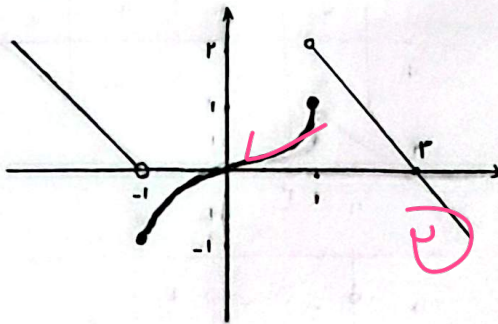
د) $\cot(180^\circ) = \frac{1}{0} \Rightarrow$ تعریف نشده

بلبر شده، پس $y_1 = \pm y_2$ نمی تواند باشد. تابع است.

الف) $\sin \alpha = \frac{1}{\sqrt{2}} \Rightarrow \alpha: \frac{\pi}{4}, \frac{5\pi}{4}$

ب) $\cos \alpha = \frac{\sqrt{2}}{\sqrt{2}} \Rightarrow \alpha: \frac{\pi}{4}, \frac{7\pi}{4}$



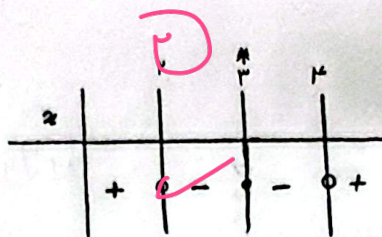


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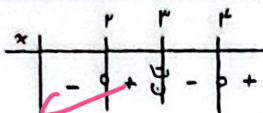
$$u) y = \overset{1}{(x-1)} \overset{2}{(x-2)} \overset{3}{(x-4)} = x$$

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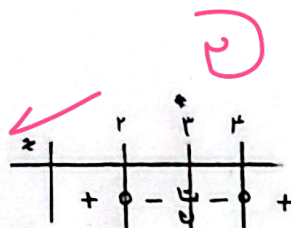
$$y = \overset{1}{(x-1)} \overset{2}{(x-2)} \overset{3}{(x-4)} =$$



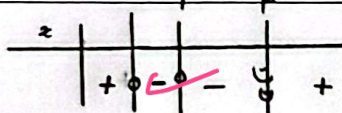
$$u) y = \frac{\overset{1}{(x-1)} \overset{2}{(x-4)}}{\overset{3}{(x-2)}} =$$



$$b) y = \frac{\overset{1}{(x-1)} \overset{2}{(x-4)}}{\overset{3}{(x-2)} \overset{4}{(x-2)}} =$$

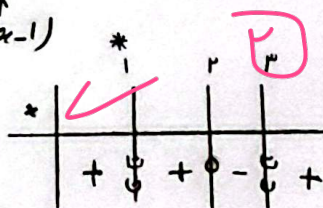


$$d) y = \frac{x^2 - 3x + 2}{x^2 - 2} =$$

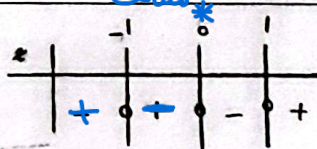


$$(x^2 - 3x + 2) + (x-1) = x^2 + x - 2 \Rightarrow (x+2)(x-1)$$

$$b) \frac{x^2 - 3x + 2}{x^2 - 2x + 2} = \frac{\overset{1}{(x-1)} \overset{2}{(x-2)}}{\overset{3}{(x-2)} \overset{4}{(x-1)}} \Rightarrow$$



$$u) y = \frac{x^2 - x^2}{x^2 + 2x + 2} = \frac{z^2 (x+1)(x-1)}{x^2 + 2x + 2}$$



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چون $\Delta = 1 - 4 = -3 < 0$ پس هیچ ریشه حقیقی ندارد $x \in \mathbb{R}$

$$b) y = \frac{x^2 + 2x + 2}{x^2 + 2x + 2} =$$

چون $\Delta < 0$ ریشه حقیقی ندارد $x \in \mathbb{R}$ معرّف

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پس عبارت در انزاعا هر عددی + حاصله بود.

ب- 1) $\int y_1^3 + y_1 = \frac{y_1^4}{4} - 1$ $\xrightarrow{y_1 = x_1}$ $y_1^3 + y_1 = y_2^3 + y_2 \rightarrow \underbrace{y_1^3 - y_2^3}_{\text{فیل و فیلان}} + (y_1 - y_2) = 0$

$$(y_1 - y_r)(y_1^2 + y_1 y_r + y_r^2) + (y_1 - y_r)z \rightarrow (y_1 - y_r)(y_1^2 + y_1 y_r + y_r^2 + 1) = 0$$

$$y_1 = \frac{-y_2 \pm \sqrt{y_2^2 - 4(y_2 + 1)}}{2}$$

الف - ١٠ $y = \frac{n^2 - n^2}{n^2 + n + 2} = \frac{n^2(n^2 - 1)}{n^2 + n + 2} = \frac{n^2(n-1)(n+1)}{n^2 + n + 2}$

صفر منفرد ←
صفر منفرد!
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Diagram: A number line with points marked -1, 0, 1. The points are labeled with signs: +, -, +. The point 0 is marked with a red asterisk (*).