

۲. آفرین عالی بود

الف) $y = \sqrt{\frac{x^2-4x}{x^2-x}} \rightarrow \frac{x^2-4x}{x^2-x} \geq 0 \rightarrow \frac{x(x-4)}{x(x-1)} \geq 0 \rightarrow \frac{x-4}{x-1} \geq 0$
 جدول علامت: $\begin{array}{c|ccc} & -1 & 0 & 1 & 4 \\ \hline & + & - & + & - \end{array}$
 $Df = (-1, 0) \cup (0, 1) \cup [4, +\infty)$

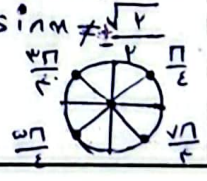
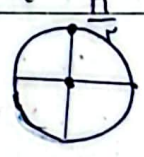
ب) $y = \sqrt{\frac{x^4-x^3}{x^3-x}} \rightarrow \frac{x^4-x^3}{x^3-x} \geq 0 \rightarrow \frac{x^3(x-1)}{x^2(x-1)} \geq 0 \rightarrow \frac{x^3}{x^2} \geq 0 \rightarrow x \geq 0$
 $Df = (-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, +\infty)$
 $Df = \mathbb{R} - \{-1, 0, 1\}$

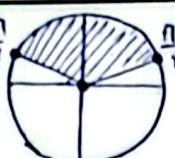
الف) $y = \frac{x+1}{\sqrt{x+1}}$
 $x+1 \geq 0 \rightarrow x \geq -1$
 $Df = \mathbb{R}^+ \cup Df = (0, +\infty)$
 ب) $y = \frac{1}{\sqrt{x+1}}$
 $x+1 \geq 0 \rightarrow x \geq -1$
 $Df = \mathbb{R}^+ \cup Df = (0, +\infty)$

الف) $y = \sqrt{2-[x]} \rightarrow 2-[x] \geq 0 \rightarrow [x] \leq 2 \Rightarrow Df = (-\infty, 3)$
 ب) $y = \frac{1}{\sqrt{2-[x]}}$
 $2-[x] > 0 \rightarrow [x] < 2 \Rightarrow Df = (-\infty, 2)$

الف) $y = \sqrt{[x-\frac{1}{p}] + [x+\frac{1}{p}]} \rightarrow [x-\frac{1}{p}] + [x+\frac{1}{p}] \geq 0 \Rightarrow [x-\frac{1}{p}] + [x-\frac{1}{p}+1] \geq 0$
 $= [x-\frac{1}{p}] + [x-\frac{1}{p}] + 1 \geq 0 \Rightarrow 2[x-\frac{1}{p}] \geq -1 \Rightarrow [x-\frac{1}{p}] \geq -\frac{1}{2}$
 $= x \geq \frac{1}{p} \Rightarrow Df = [\frac{1}{p}, +\infty)$
 ب) $y = \sqrt{[x-\frac{1}{p}] + [x+\frac{1}{p}]} \rightarrow [x-\frac{1}{p}] + [x+\frac{1}{p}] \geq 0 \Rightarrow [x-\frac{1}{p}] + [-(x-\frac{1}{p})] \geq 0$
 $\text{if } \begin{cases} a \in \mathbb{Z} & [a] + [a] = 0 \checkmark \\ a \notin \mathbb{Z} & [a] + [-a] = -1 \times \end{cases} \Rightarrow x - \frac{1}{p} \in \mathbb{Z} \rightarrow x \in \mathbb{Z} + \frac{1}{p} \Rightarrow Df = \{x | x = k + \frac{1}{p}, k \in \mathbb{Z}\}$

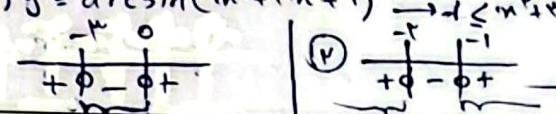
الف) $y = \frac{2}{\sin x - 1} \rightarrow \sin x - 1 \neq 0 \rightarrow \sin x \neq 1$
 $\Rightarrow Df = \mathbb{R} - \{2k\pi + \frac{\pi}{2}\}$
 ب) $y = \frac{3}{2\sin^2 x - 1} \rightarrow 2\sin^2 x - 1 \neq 0 \rightarrow 2\sin^2 x \neq 1 \rightarrow \sin^2 x \neq \frac{1}{2} \rightarrow \sin x \neq \pm \frac{\sqrt{2}}{2}$
 $\Rightarrow Df = \mathbb{R} - \{ \frac{k\pi}{2} \pm \frac{\pi}{4} \}$



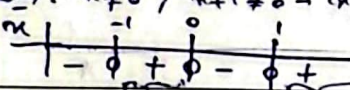
الف) $y = \sqrt{x \sin x - 1} \rightarrow x \sin x - 1 \geq 0 \rightarrow x \sin x \geq 1 \rightarrow \sin x \geq \frac{1}{x}$ 
 $Df = [2k\pi + \frac{\pi}{4}, 2k\pi + \frac{\omega\pi}{4}]$

ب) $y = \sqrt{\cos x - 1} \rightarrow \cos x - 1 \geq 0 \rightarrow \cos x \geq 1$
 $\Rightarrow Df = \{2k\pi\} \cup Df = \{2k\pi | k \in \mathbb{Z}\}$ 
 * توجه داریم که مقادیر $\cos x$ در بازه $[-1, 1]$ است پس $\cos x$ تنها می تواند برابر با 1 باشد.

الف) $\cos y = |x| - 3 \rightarrow -1 \leq |x| - 3 \leq 1 \rightarrow -2 \leq |x| \leq 4 \rightarrow -4 \leq x \leq 4$
 $\Rightarrow Df = [-4, -2] \cup [2, 4]$

* توجه داریم که مقادیر $|x| - 3$ تنها در بازه $[-1, 1]$ قرار داشته باشند.
 * توجه داریم که $\arcsin$ و $\arccos$ عددی در بازه $[-1, 1]$ است.
 ب) $y = \arcsin(x^2 + 2x + 1) \rightarrow -1 \leq x^2 + 2x + 1 \leq 1$
 $\rightarrow x^2 + 2x \leq 0 \rightarrow x(x+2) \leq 0 \rightarrow -2 \leq x \leq 0$
 $\rightarrow x^2 + 2x + 2 \geq 0 \rightarrow (x+1)^2 + 1 \geq 0$ (همیشه درست است)
 $\Rightarrow Df = [-2, 0] \cup [-1, 0]$ 

الف) $y = \sqrt{x \log_3(x-2)} \rightarrow \begin{cases} x-2 > 0 \rightarrow x > 2 \\ x \log_3(x-2) \geq 0 \end{cases}$
 $\rightarrow x < x \leq 11 \rightarrow Df = [2, 11]$
 $\log_3(x-2) \leq 2 \rightarrow x-2 \leq 9 \rightarrow x \leq 11$

ب) $y = \log(x^3 - x) \rightarrow x^3 - x > 0 \rightarrow x(x^2 - 1) > 0 \rightarrow x(x+1)(x-1) > 0$
 * توجه داریم که $x \neq 0, x+1 \neq 0, x-1 \neq 0$
 $\rightarrow Df = (-1, 0) \cup (1, +\infty)$ 

الف) $y = \frac{x}{x^2 + 1} \rightarrow x^2 + 1 \neq 0 \rightarrow x^2 \neq -1$
 $Df = \mathbb{R}$

ب) $y = \frac{x}{x^2 - 1} \rightarrow x^2 - 1 \neq 0 \rightarrow x^2 \neq 1 \rightarrow x \neq \pm 1 \rightarrow Df = \mathbb{R} - \{0\}$

ج) $y = \frac{x}{x^2 - 2} \rightarrow x^2 - 2 \neq 0 \rightarrow x^2 \neq 2 \rightarrow x \neq \pm \sqrt{2} \rightarrow Df = \mathbb{R} - \{\frac{1}{\sqrt{2}}\}$

د) $y = \frac{x}{x^2 - 3} \rightarrow x^2 - 3 \neq 0 \rightarrow x^2 \neq 3 \rightarrow x \neq \pm \sqrt{3} \rightarrow Df = \mathbb{R} - \{\log \sqrt{3}\}$

الف) $y = (x+1)! \rightarrow x+1 \in \mathbb{N} \rightarrow x \in \mathbb{N} - 1 \rightarrow x \in \mathbb{N} - 1$
 $\Rightarrow Df = \{x | x = k-1, k \in \mathbb{N}\}$

ب) $y = \left(\frac{x-2}{x-5}\right)! \rightarrow \frac{x-2}{x-5} \in \mathbb{N}$
 $\rightarrow \frac{x-2}{x-5} = k \rightarrow x-2 = k(x-5) \rightarrow x-2 = kx-5k \rightarrow x-kx = -5k+2 \rightarrow x(1-k) = -5k+2$
 $\rightarrow x = \frac{-5k+2}{1-k} \rightarrow Df = \{x | x = \frac{-5k+2}{1-k}, k \in \mathbb{N}\}$
 * توجه داریم که $\frac{x-2}{x-5} \in \mathbb{N}$ 