

1) $y = \sqrt{\frac{x^2 - 4x}{x^3 - x}} \geq 0 \Rightarrow \frac{x(x-4)}{x(x^2-1)} \geq 0$

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$\frac{x-4}{(x-1)(x+1)}$ $\begin{array}{c|ccc} -1 & * & 0 & 1 & 4 \\ \hline & + & - & + & - \end{array}$ $(-1, 0)$ $(0, 1)$ $(1, 4)$ $(4, +\infty)$

$D = (-1, 0) \cup (0, 1) \cup [4, +\infty)$

2) $y = \sqrt{\frac{x^3 - x^2}{x^2 - x}} = \frac{x^2(x-1)}{x(x^2-1)} \geq 0$ $\begin{array}{c|ccc} * & -1 & * & 0 & * & 1 \\ \hline & + & - & + & - & + \end{array}$

$D = \mathbb{R} \setminus \{-1, 0, 1\}$

3) $y = \frac{x+1}{\sqrt{x+1}} = \sqrt{x+1} > 0 \Rightarrow x+1 > 0 \Rightarrow x > -1$

$D = (-1, +\infty)$

4) $y = \frac{1}{\sqrt{x|x|}} = \frac{1}{\sqrt{x^2}} = \frac{1}{|x|} > 0 \Rightarrow x \neq 0$

$D = (-\infty, 0) \cup (0, +\infty)$

5) $y = \sqrt{2 - [x]} \geq 0 \Rightarrow [x] \leq 2$

$D = (-\infty, 3)$

$[x] \leq 2 \Rightarrow x < 3$

6) $y = \sqrt{2 - [x]} \geq 0 \Rightarrow [x] \leq 2 \Rightarrow [x] \leq 1 \rightarrow x < 2$

$D = (-\infty, 2)$

7) $y = \sqrt{[x - \frac{1}{x}] + [x + \frac{1}{x}]}$

$[x + \frac{1}{x}] = [x - \frac{1}{x} + 1] = [x - \frac{1}{x}] + 1$

$2[x - \frac{1}{x}] + 1 \geq 0 \Rightarrow [x - \frac{1}{x}] \geq -\frac{1}{2}$

$[x - \frac{1}{x}] \geq 0 \Rightarrow x - \frac{1}{x} \geq 0 \Rightarrow x \geq \frac{1}{x}$

8) $y = \sqrt{[x - \frac{1}{x}] + [-x + \frac{1}{x}]}$

$t = x - \frac{1}{x} \quad [t] + [-t] = \begin{cases} 0 & t \in \mathbb{Z} \\ -1 & t \notin \mathbb{Z} \end{cases}$

$x - \frac{1}{x} = k \in \mathbb{Z} \Rightarrow x = k + \frac{1}{x}$

$D = \{k + \frac{1}{x} \mid k \in \mathbb{Z}\}$

ا) $y = \frac{r}{\sin \alpha - 1} = \sin \alpha - 1 \neq 0 \Rightarrow \sin \alpha \neq 1 \Rightarrow \alpha \neq k\pi + \frac{\pi}{2} \quad (k \in \mathbb{Z})$

$D_f = \mathbb{R} \setminus \left\{ k\pi + \frac{\pi}{2} \mid k \in \mathbb{Z} \right\}$

ب) $y = \frac{r}{r \sin^2 \alpha - 1} = r \sin^2 \alpha - 1 \neq 0 \Rightarrow \cos^2 \alpha \neq 0 \Rightarrow \cos \alpha \neq 0 \Rightarrow \alpha \neq k\pi + \frac{\pi}{2} \Rightarrow$

$\alpha \neq \frac{k\pi}{2} + \frac{\pi}{2} \quad (k \in \mathbb{Z}) \quad D_f = \mathbb{R} \setminus \left\{ \frac{k\pi}{2} + \frac{\pi}{2} \mid k \in \mathbb{Z} \right\}$

ا) $y = \sqrt{r \sin \alpha - 1} \Rightarrow r \sin \alpha - 1 \geq 0 \Rightarrow \sin \alpha \geq \frac{1}{r}$

$D_f = \bigcup_{k \in \mathbb{Z}} \left[k\pi + \frac{\pi}{4}, k\pi + \frac{3\pi}{4} \right]$

ب) $y = \sqrt{\cos \alpha - 1} \Rightarrow \cos \alpha - 1 \geq 0 \Rightarrow \cos \alpha \geq 1 \quad \cos \alpha \leq 1 \quad \cos \alpha = 1$
 $D_f = \{ k\pi \mid k \in \mathbb{Z} \}$

ا) $\cos y = |x| - 3 \quad -1 \leq |x| - 3 \leq 1 \Rightarrow 2 \leq |x| \leq 4$

$D = [-4, -2] \cup [2, 4]$

ب) $y = \arcsin(x^2 + 3x + 1) \Rightarrow -1 \leq x^2 + 3x + 1 \leq 1$

① $x^2 + 3x + 1 \geq -1 \Rightarrow x^2 + 3x + 2 \geq 0 \Rightarrow (x+1)(x+2) \geq 0 \Rightarrow x \in (-\infty, -2] \cup [-1, +\infty)$

② $x^2 + 3x + 1 \leq 1 \Rightarrow x^2 + 3x \leq 0 \Rightarrow x(x+3) \leq 0 \Rightarrow x \in [-3, 0]$

$D_f = [-3, -2] \cup [-1, 0]$

ا) $y = \sqrt{2 - \log_3(x-2)} = x - 2 > 0 \Rightarrow x > 2$

$2 - \log_3(x-2) \geq 0 \Rightarrow \log_3(x-2) \leq 2 \Rightarrow x-2 \leq 3^2 = 9 \Rightarrow x \leq 11$

$D_f = (2, 11]$

ب) $y = \log(x^3 - x) \Rightarrow x^3 - x > 0 \Rightarrow x(x-1)(x+1) > 0$

$D_f = (-1, 0) \cup (1, +\infty)$

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$\begin{array}{c} -1 \quad 0 \quad 1 \\ \hline - \quad + \quad - \quad + \end{array}$

ا) $y = \frac{x}{x^u + 1} \Rightarrow x^u > 0 \quad u \in \mathbb{R} \quad D_f = \mathbb{R}$ (4)

ب) $y = \frac{x}{x^u - 1} \Rightarrow x^u - 1 \neq 0 \Rightarrow x^u \neq 1 \Rightarrow u \neq 0$ (2)

$D_f = \mathbb{R} \setminus \{0\}$

ج) $x^u - x \neq 0 \Rightarrow x^u \neq x \Rightarrow x^u \neq 1 \Rightarrow u \neq \frac{1}{x}$

$D_f = \mathbb{R} \setminus \left\{ \frac{1}{x} \right\}$

د) $y = \frac{x}{x^u - x} \Rightarrow x^u - x \neq 0 \Rightarrow x^u \neq x \Rightarrow u \neq \log_x 3 \quad D_f = \mathbb{R} \setminus \left\{ \log_x 3 \right\}$

ا) $y = (kx+1)! \quad \exists a, b, c = k \quad (k \in \{0, 1, 2, \dots\}) \Rightarrow u = \frac{k-1}{x}$ (1)

$D_f = \left\{ \frac{k-1}{x} \mid k \in \mathbb{W} \right\} = \left\{ -\frac{1}{x}, 0, \frac{1}{x}, \frac{2}{x}, \dots \right\}$ (2)

ب) $y = \left(\frac{kx-r}{kx-\delta} \right)! \quad \frac{kx-r}{kx-\delta} = k \quad (k \in \mathbb{W})$

$kx-r = kx-\delta \Rightarrow u = \frac{\delta k - r}{kx - r}$

$D_f = \left\{ \frac{\delta k - r}{kx - r} \mid k \in \mathbb{W} \right\}$

~~$kx-r$~~