

الف) $y = \sqrt{\frac{x^2-2x}{x^3-x}} \rightarrow \sqrt{\frac{x(x-2)}{x(x+1)(x-1)}} > 0 \rightarrow \frac{-1}{-} \frac{0}{0} \frac{2}{+} \frac{1}{+} \frac{1}{+} \frac{2}{+}$

$D_f = (-1, 0) \cup (0, 1) \cup [2, +\infty)$

ب) $y = \sqrt{\frac{x^3-x^2}{x^3-x}} \rightarrow \sqrt{\frac{x^2(x+1)(x-1)}{x(x+1)(x-1)}} > 0 \rightarrow \frac{-1}{0} \frac{0}{-} \frac{2}{+} \frac{1}{+} \frac{1}{+}$
 $D_f = \mathbb{R}^+ - \{0, \pm 1\}$

الف) $y = \frac{x+1}{\sqrt{x+1}}$ $\rightarrow x+1 > 0 \rightarrow x > -1 \rightarrow D_f = \mathbb{R}^+ \cup (-1, +\infty)$

ب) $y = \frac{1}{\sqrt{x|x|}}$ $\rightarrow x|x| > 0 \rightarrow D_f = \mathbb{R}^+ \cup (-\infty, -1)$

الف) $y = \sqrt{2-[x]}$ $\rightarrow 2-[x] > 0 \rightarrow [x] \leq 2 \rightarrow x < 3 \rightarrow D_f = (-\infty, 3)$

ب) $y = \frac{1}{\sqrt{2-[x]}}$ $\rightarrow 2-[x] > 0 \rightarrow [x] < 2 \rightarrow x < 2 \rightarrow D_f = (-\infty, 2)$

الف) $\sqrt{[x-\frac{1}{2}]} + [x+\frac{1}{2}] > 0 \rightarrow [x-\frac{1}{2}] + [x+\frac{1}{2}] > 0 \rightarrow [x-\frac{1}{2}+1] + [x+\frac{1}{2}] > 0$
 $\rightarrow [x+\frac{1}{2}] + [x+\frac{1}{2}] > 0 \rightarrow 2[x+\frac{1}{2}] > 0 \rightarrow [x+\frac{1}{2}] > 0 \rightarrow x+\frac{1}{2} > 0$ (که این را می توانیم بگوییم عدد صحیح بزرگتر از $\frac{1}{2}$ است)
 $\rightarrow x > -\frac{1}{2} \rightarrow D_f = [-\frac{1}{2}, +\infty)$

ب) $y = \sqrt{[x-\frac{1}{2}] + [-x+\frac{1}{2}]} \rightarrow \sqrt{[x-\frac{1}{2}] + [-(x-\frac{1}{2})]} \xrightarrow{\text{if } a \in \mathbb{Z} \quad [a] + [-a] = 0 \rightarrow x-\frac{1}{2} \in \mathbb{Z}}$
 $\xrightarrow{\text{if } a \notin \mathbb{Z} \quad [a] + [-a] = -1}$

الف) $y = \frac{2}{\sin x - 1}$ $\rightarrow \sin x \neq 1$ $\rightarrow D_f = \mathbb{R} - \{2k\pi + \frac{\pi}{2}\}$

ب) $y = \frac{2}{\sqrt{\sin x} - 1}$ $\rightarrow \sqrt{\sin x} \neq 1 \rightarrow \sin x \neq \frac{1}{4} \rightarrow \sin x \neq \pm \frac{\sqrt{2}}{2}$ $\rightarrow D_f = \mathbb{R} - \{k\pi \pm \frac{\pi}{4}\}$

الف) $y = \sqrt{r \sin x - 1} \rightarrow r \sin x - 1 \geq 0 \rightarrow r \sin x \geq 1 \rightarrow \sin x \geq \frac{1}{r}$ 
 $D_f = \left[r k \pi + \frac{\pi}{r}, r k \pi + \frac{3\pi}{r} \right]$

ب) $y = \sqrt{\cos x - 1} \rightarrow \cos x - 1 \geq 0 \rightarrow \cos x \geq 1$ 
 $D_f = \{ r k \pi \}$

الف) $\cos y = |u| - r \rightarrow -1 \leq |u| - r \leq 1 \rightarrow r \leq |u| \leq r+1 \rightarrow r \leq |u| \leq r+1$
 $D_f = [-r, -r+1] \cup [r, r+1]$

ب) $y = \arcsin(u^r + r u + 1)$
 $D_f = [-r, -r+1] \cup [-1, 0]$

الف) $y = \sqrt{r - \log_n(n-r)} \rightarrow r - \log_n(n-r) \geq 0 \rightarrow \log_n(n-r) \leq r \rightarrow n-r \leq n^r \rightarrow n \leq n^r$
 $D_f = (r, 11]$

ب) $\log(n^r - n) \rightarrow n^r - n > 0 \rightarrow n(n+1)(n-1) > 0 \rightarrow \frac{-1 \pm \sqrt{1+4}}{2} \rightarrow D_f = (-1, 0) \cup (1, +\infty)$

الف) $y = \frac{r}{e^x + 1} \rightarrow e^x \neq -1 \rightarrow D_f = \mathbb{R}$

ب) $y = \frac{r}{e^x - 1} \rightarrow e^x \neq 1 \rightarrow x \neq 0 \rightarrow D_f = \mathbb{R} - \{0\}$

ج) $y = \frac{r}{e^x - r} \rightarrow e^x - r \neq 0 \rightarrow e^x \neq r \rightarrow x \neq \log_e r \rightarrow D_f = \mathbb{R} - \{\log_e r\}$

د) $y = \frac{r}{e^x - r} \rightarrow e^x - r \neq 0 \rightarrow e^x \neq r \rightarrow x \neq \log_e r \rightarrow D_f = \mathbb{R} - \{\log_e r\}$

الف) $y = (r x + 1)! \rightarrow r x + 1 \in \mathbb{N} \rightarrow x \in \frac{\mathbb{N} - 1}{r} \rightarrow D_f = \left\{ x \mid x = \frac{k-1}{r}, k \in \mathbb{N} \right\}$

ب) $y = \left(\frac{r x - r}{r x - a} \right)! \rightarrow \frac{r x - r}{r x - a} \in \mathbb{N} \rightarrow x = \frac{r - a \mathbb{N}}{r - r \mathbb{N}} = \frac{r - a k}{r - r k} \rightarrow D_f = \left\{ x \mid x = \frac{r - a k}{r - r k}, k \in \mathbb{N} \right\}$

$$n = - \frac{dy - b}{cy - a}$$

$$\begin{aligned} (a, b): r x - r &= r x \mathbb{N} - a \mathbb{N} \\ r x - r x \mathbb{N} &= r - a \mathbb{N} \\ n(r - r \mathbb{N}) &= r - a \mathbb{N} \rightarrow n = \frac{r - a \mathbb{N}}{r - r \mathbb{N}} \end{aligned}$$