

الف) $y = \sqrt{\frac{x^2-4}{x^3-x}} \rightarrow \sqrt{\frac{x(x-4)}{x(x+1)(x-1)}} > 0 \rightarrow \frac{-1}{0} \frac{0}{-1} \frac{1}{1} \frac{4}{1}$

$D_f = (-1, 0) \cup (0, 1) \cup [4, +\infty)$

ب) $y = \sqrt{\frac{x^3-x^2}{x^3-x}} \rightarrow \sqrt{\frac{x^2(x+1)(x-1)}{x(x+1)(x-1)}} > 0 \rightarrow \frac{-1}{0} \frac{0}{-1} \frac{1}{1} \frac{1}{1}$
 $D_f = \mathbb{R} - \{0, \pm 1\}$

الف) $y = \frac{x+1}{\sqrt{x+1}}$
 $\begin{matrix} 0 & x \\ + & x \\ - & x \end{matrix}$
 $\rightarrow x+1 > 0 \rightarrow x > -1 \rightarrow D_f = \mathbb{R}^+ \cup (0, +\infty)$

ب) $y = \frac{1}{\sqrt{x|x|}}$
 $\begin{matrix} 0 & x \\ + & x \\ - & x \end{matrix}$
 $\rightarrow x|x| > 0 \rightarrow D_f = \mathbb{R}^+ \cup (0, +\infty)$

الف) $y = \sqrt{2-[x]}$
 $\rightarrow 2-[x] > 0 \rightarrow [x] \leq 2 \rightarrow x < 3 \rightarrow D_f = (-\infty, 3)$

ب) $y = \frac{1}{\sqrt{2-[x]}}$
 $\rightarrow 2-[x] > 0 \rightarrow [x] < 2 \rightarrow x < 2 \rightarrow D_f = (-\infty, 2)$

الف) $\sqrt{[x-\frac{1}{2}]} + [x+\frac{1}{2}] > 0 \rightarrow [x-\frac{1}{2}] + [x+\frac{1}{2}] > 0 \rightarrow [x-\frac{1}{2}+1] + [x+\frac{1}{2}] > 0$
 $\rightarrow [x+\frac{1}{2}] - 1 + [x+\frac{1}{2}] > 0 \rightarrow 2[x+\frac{1}{2}] > 1 \rightarrow [x+\frac{1}{2}] > \frac{1}{2}$
 $\rightarrow x > \frac{1}{4} \rightarrow D_f = [\frac{1}{4}, +\infty)$
 (توجه: $\frac{1}{4}$ عدد صحیح بعد از $\frac{1}{2}$ است)

ب) $y = \sqrt{[x-\frac{1}{2}] + [-x+\frac{1}{2}]} \rightarrow \sqrt{[x-\frac{1}{2}] + [-(x-\frac{1}{2})]}$
 اگر $a \in \mathbb{Z}$ $[a] + [-a] = 0 \rightarrow x-\frac{1}{2} \in \mathbb{Z}$
 اگر $a \notin \mathbb{Z}$ $[a] + [-a] = -1$

الف) $y = \frac{2}{\sin x - 1} \rightarrow \sin x \neq 1$

 $\rightarrow D_f = \mathbb{R} - \{2k\pi + \frac{\pi}{2}\}$
 $\rightarrow D_f = \{x | x = k + \frac{1}{2}, k \in \mathbb{Z}\}$

ب) $y = \frac{2}{\sqrt{\sin x} - 1} \rightarrow \sqrt{\sin x} \neq 1 \rightarrow \sin x \neq \frac{1}{4} \rightarrow \sin x \neq \pm \frac{\sqrt{2}}{2}$

 $\rightarrow D_f = \mathbb{R} - \{k\pi \pm \frac{\pi}{6}\}$

الف) $y = \sqrt{x \sin x - 1} \rightarrow x \sin x - 1 \geq 0 \rightarrow x \sin x \geq 1 \rightarrow \sin x \geq \frac{1}{x}$ 
 $D_f = \left[2k\pi + \frac{\pi}{2}, 2k\pi + \frac{3\pi}{2} \right]$

ب) $y = \sqrt{\cos x - 1} \rightarrow \cos x - 1 \geq 0 \rightarrow \cos x \geq 1$ 
 $D_f = \{2k\pi\}$

الف) $\cos y = |x| - 2 \rightarrow -1 \leq |x| - 2 \leq 1 \rightarrow -1 \leq |x| \leq 3 \rightarrow |x| \leq 3$
 $D_f = [-3, 3]$

ب) $y = \arcsin(x^2 + 2x + 1)$
 $-1 \leq x^2 + 2x + 1 \leq 1 \rightarrow 0 \leq x^2 + 2x \leq 0 \rightarrow x(x+2) \geq 0 \rightarrow x \leq -2 \text{ or } x \geq 0$
 $D_f = (-\infty, -2] \cup [0, +\infty)$

الف) $y = \sqrt{x - \log_e(x-2)}$
 $x - 2 \geq 0 \rightarrow x \geq 2$
 $x - \log_e(x-2) \geq 0 \rightarrow \log_e(x-2) \leq x \rightarrow x-2 \leq e^x \rightarrow x \leq 11$
 $D_f = [2, 11]$

ب) $\log(x^2 - x)$
 $x^2 - x > 0 \rightarrow x(x-1) > 0 \rightarrow x < 0 \text{ or } x > 1$
 $D_f = (-\infty, 0) \cup (1, +\infty)$

الف) $y = \frac{x}{e^x + 1}$
 $e^x \neq -1$
 $D_f = \mathbb{R}$

ب) $y = \frac{x}{e^x - 1}$
 $e^x \neq 1 \rightarrow x \neq 0$
 $D_f = \mathbb{R} - \{0\}$

ج) $y = \frac{x}{e^x - 2}$
 $e^x - 2 \neq 0 \rightarrow e^x \neq 2 \rightarrow x \neq \log_e 2$
 $D_f = \mathbb{R} - \{\log_e 2\}$

د) $y = \frac{x}{e^x - 3}$
 $e^x - 3 \neq 0 \rightarrow e^x \neq 3 \rightarrow x \neq \log_e 3$
 $D_f = \mathbb{R} - \{\log_e 3\}$

الف) $y = (x+1)!$
 $x+1 \in \mathbb{N} \rightarrow x \in \mathbb{N} - 1 \rightarrow x \in \frac{n-1}{1}$
 $D_f = \{x | x = \frac{n-1}{1}, n \in \mathbb{N}\}$

ب) $y = \left(\frac{x-2}{x-5}\right)!$
 $\frac{x-2}{x-5} \in \mathbb{N} \rightarrow x = \frac{2-5n}{1-5n} = \frac{2-5n}{1-5n} \rightarrow D_f = \{x | x = \frac{2-5n}{1-5n}, n \in \mathbb{N}\}$
 $x = -\frac{dy-b}{cy-a}$
 $(\omega): x-2 = 2n-5n \rightarrow x-2 = -3n \rightarrow x = -3n+2$
 $x-5 = -5n+5 \rightarrow x-5 = -5n+5 \rightarrow x = -5n+10$
 $x(x-5) = (-3n+2)(-5n+10) \rightarrow x = \frac{2-5n}{1-5n}$