

مأ آفرین

الف) $y = \sqrt{\frac{x^2-4x}{x^3-x}} \Rightarrow \begin{cases} x^2-4x \geq 0 \rightarrow x(x-4) \geq 0 \rightarrow x=4, 0 \text{ و } x=-1, 0 \text{ و } x=1, 0 \end{cases}$
 $\frac{-1}{-} \frac{0}{+} \frac{1}{+} \frac{4}{+} \Rightarrow Df = (-1, 0) \cup (0, 1) \cup [4, +\infty)$

ب) $y = \sqrt{\frac{x^3-x^2}{x^3-x}} \Rightarrow \begin{cases} x^3-x^2 \geq 0 \Rightarrow x^2(x-1) \geq 0 \Rightarrow x=1, 0 \text{ و } x=-1, 0 \text{ و } x=1, 0 \end{cases}$
 $\frac{-1}{+} \frac{0}{+} \frac{1}{+} \frac{4}{+} \Rightarrow Df = (-\infty, -1) \cup (-1, 0) \cup (1, +\infty)$

الف) $y = \frac{x+1}{\sqrt{x+|x|}} \Rightarrow x+|x| > 0 \Rightarrow |x| > -x \Rightarrow Df = \mathbb{R}^+$

ب) $y = \frac{1}{\sqrt{x|x|}} \Rightarrow x|x| > 0 \Rightarrow Df = (0, +\infty) \subset \mathbb{R}^+$

الف) $y = \sqrt{2-[x]} \Rightarrow 2-[x] \geq 0 \Rightarrow [x] \leq 2 \rightarrow x < 3 \Rightarrow Df = (-\infty, 3)$

ب) $y = \frac{1}{\sqrt{2-[x]}} \Rightarrow 2-[x] > 0 \Rightarrow [x] < 2 \Rightarrow x < 2 \Rightarrow Df = (-\infty, 2)$

الف) $y = \sqrt{[x-\frac{1}{p}] + [x+\frac{p}{p}]} \Rightarrow [x-\frac{1}{p}] + [x+\frac{p}{p}] \geq 0 \Rightarrow [x+\frac{p}{p}-1] + [x+\frac{p}{p}] \geq 0$
 $[x+\frac{p}{p}-1] + [x+\frac{p}{p}] \Rightarrow 2[x+\frac{p}{p}]-1 \geq 0 \Rightarrow 2[x+\frac{p}{p}] \geq 1 \Rightarrow [x+\frac{p}{p}] \geq \frac{1}{2} \Rightarrow x+\frac{p}{p} \geq 1$
 $\Rightarrow x \geq \frac{1}{p} \Rightarrow Df = [\frac{1}{p}, +\infty)$

ب) $y = \sqrt{[x-\frac{1}{p}] + [-(x-\frac{1}{p})]} \xrightarrow{\text{if}} \begin{cases} a \in \mathbb{Z} & [a] + [-a] = 0 \\ a \in \mathbb{Z} & [a] + [-a] = -1 \end{cases} \Rightarrow x-\frac{1}{p} \in \mathbb{Z}$
 $\Rightarrow Df = \{x | x = k + \frac{1}{p} \text{ و } k \in \mathbb{Z}\}$

الف) $y = \frac{r}{\sin x - 1} \Rightarrow \sin x - 1 \neq 0 \Rightarrow \sin x \neq 1$

$Df = \mathbb{R} - \{2k\pi + \frac{\pi}{2}\}$



ب) $y = \frac{r}{r \sin^2 x - 1} \Rightarrow r \sin^2 x - 1 \neq 0 \Rightarrow r \sin^2 x \neq 1 \Rightarrow \sin^2 x \neq \frac{1}{r}$

$\Rightarrow \sin x \neq \pm \frac{1}{\sqrt{r}} \Rightarrow \sin x \neq \pm \frac{\sqrt{r}}{r}$

$Df = \mathbb{R} - \{k\pi \pm \frac{\pi}{r}\}$



الف) $y = \sqrt{y \sin x - 1} \Rightarrow y \sin x - 1 \geq 0 \Rightarrow y \sin x \geq 1 \Rightarrow \sin x \geq \frac{1}{y}$
 $D_f = [2k\pi + \frac{\pi}{y}, 2k\pi + \pi - \frac{\pi}{y}]$



ب) $\sqrt{\cos x - 1} \Rightarrow \cos x - 1 \geq 0 \Rightarrow \cos x \geq 1$

$D_f = \{2k\pi | k \in \mathbb{Z}\}$

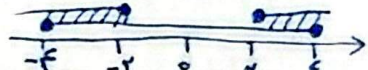


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الف) $\cos y = |x| - 3 \Rightarrow -1 \leq \cos y \leq 1 \Rightarrow -1 \leq |x| - 3 \leq 1 \Rightarrow -1 + 3 \leq |x| \leq 1 + 3$
 $\Rightarrow 2 \leq |x| \leq 4$

$|x| \geq 2$ ناحه $\rightarrow x \geq 2$ و $x \leq -2$
 $|x| \leq 4$ ناحه $\rightarrow -4 \leq x \leq 4$

$D_f = [-4, -2] \cup [2, 4]$



ب) $y = \arcsin(x^2 + 3x + 1) \Rightarrow -1 \leq x^2 + 3x + 1 \leq 1 \rightarrow \begin{cases} x^2 + 3x + 1 \geq -1 \rightarrow x^2 + 3x + 2 \geq 0 \rightarrow \\ x^2 + 3x + 1 \leq 1 \rightarrow x^2 + 3x \leq 0 \end{cases}$

مثلا: $\rightarrow (x+2)(x+1) \geq 0$ (I) $(x+1)x \leq 0$ (II)
 $\rightarrow x(x+3) \leq 0$ (II)

$D_f = (I) \cap (II) \Rightarrow [-2, -1] \cup [-1, 0]$

الف) $y = \sqrt{2 - \log_3(x-2)} \Rightarrow 2 - \log_3(x-2) \geq 0 \rightarrow \log_3(x-2) \leq 2 \rightarrow x-2 \leq 3^2$
 $\rightarrow x-2 \leq 9 \rightarrow x \leq 11, x-2 > 0 \rightarrow x > 2$ استه اكر اين شرط $D_f = x \in (2, 11)$

ب) $y = \log(x^3 - x) \Rightarrow x^3 - x > 0 \rightarrow x(x^2 - 1) > 0 \rightarrow x(x-1)(x+1) > 0$

$\rightarrow$ ریشه ها: $-1, 0, 1$
 $D_f = (-1, 0) \cup (1, +\infty)$

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الف) $y = \frac{x}{x^2 + 1} \Rightarrow x^2 + 1 \neq 0 \Rightarrow x^2 + 1$ همیشه مثبت است $\Rightarrow D_f = \mathbb{R}$

ب) $y = \frac{x}{x^2 - 1} \Rightarrow x^2 - 1 \neq 0 \Rightarrow x^2 \neq 1 \Rightarrow x \neq \pm 1 \Rightarrow D_f = \mathbb{R} - \{1, -1\}$

ج) $x^2 - 2 \neq 0 \Rightarrow x^2 \neq 2 \Rightarrow (x^2)^{1/2} \neq 2^{1/2} \Rightarrow x^2 \neq 2 \Rightarrow x \neq \pm \sqrt{2}$
 $\Rightarrow D_f = \mathbb{R} - \{\pm \sqrt{2}\}$

د) $y = \frac{x}{x^2 - 3} \Rightarrow x^2 - 3 \neq 0 \Rightarrow x^2 \neq 3 \Rightarrow x \neq \pm \sqrt{3} \Rightarrow D_f = \mathbb{R} - \{\pm \sqrt{3}\}$

الف) $y = (x+1)! \Rightarrow x+1 \in \mathbb{N} \rightarrow x+1 = k \Rightarrow k \in \mathbb{N} \Rightarrow D_f = \{x = k-1 | k \in \mathbb{N}\}$

ب) $\frac{x^2 - 2}{x - 1} = k \rightarrow k \in \mathbb{N} \quad x - 1 \neq 0 \Rightarrow x \neq 1 \Rightarrow x \neq 1/2$

$x^2 - 2 = k(x-1) \rightarrow x^2 - 2 = kx - k \Rightarrow x^2 - kx - 2 + k = 0$
 $\Rightarrow x(x - k) = 2 - k \Rightarrow x = \frac{2 - k}{x - k}$

$D_f = \left\{ \frac{2 - k}{x - k} | k \in \mathbb{N} \right\}$
 (توجه: x و y غیری از مجموعه اعداد صحیح نیستند) $k \neq 1/2$ به شرطی که $k \neq 1/2$ به شرطی که k صحیح است