

الف) $y = \sqrt{\frac{x^2-4x}{x^3-x}} \Rightarrow \begin{cases} x^2-4x \geq 0 \Rightarrow x(x-4) \geq 0 \Rightarrow x=4, 0 \text{ و } x=-1, 0 \text{ و } 1 \\ x^3-x > 0 \Rightarrow x(x^2-1) > 0 \Rightarrow x(x+1)(x-1) > 0 \end{cases}$

$\Rightarrow Df = (-1, 0) \cup (0, 1) \cup [4, +\infty)$

۱

ب) $y = \sqrt{\frac{x^4-x^3}{x^3-x}} \Rightarrow \begin{cases} x^4-x^3 \geq 0 \Rightarrow x^3(x-1) \geq 0 \Rightarrow x^3(x-1)(x+1) \geq 0 \Rightarrow x=-1, 0 \text{ و } 1 \\ x^3-x \geq 0 \Rightarrow x(x^2-1) \geq 0 \Rightarrow x(x-1)(x+1) \geq 0 \Rightarrow x=-1, 0 \text{ و } 1 \end{cases}$

$\Rightarrow Df = (-\infty, -1) \cup (-1, 0) \cup (1, +\infty)$

۲

الف) $y = \frac{x+1}{\sqrt{x+|x|}} \Rightarrow x+|x| > 0 \Rightarrow |x| > -x \Rightarrow Df = \mathbb{R}^+$

ب) $y = \frac{1}{\sqrt{x|x|}} \Rightarrow x|x| > 0 \Rightarrow Df = (0, +\infty) \subset \mathbb{R}^+$

الف) $y = \sqrt{2-[x]} \Rightarrow 2-[x] \geq 0 \Rightarrow [x] \leq 2 \Rightarrow x < 3 \Rightarrow Df = (-\infty, 3)$

ب) $y = \frac{1}{\sqrt{2-[x]}} \Rightarrow 2-[x] > 0 \Rightarrow [x] < 2 \Rightarrow x < 2 \Rightarrow Df = (-\infty, 2)$

۳

الف) $y = \sqrt{\left[x - \frac{1}{x}\right] + \left[x + \frac{1}{x}\right]} \Rightarrow \left[x - \frac{1}{x}\right] + \left[x + \frac{1}{x}\right] \geq 0 \Rightarrow \left[x + \frac{1}{x} - 1\right] + \left[x + \frac{1}{x}\right] \geq 0$

$\left[x + \frac{1}{x}\right] - 1 + \left[x + \frac{1}{x}\right] \Rightarrow 2\left[x + \frac{1}{x}\right] - 1 \geq 0 \Rightarrow 2\left[x + \frac{1}{x}\right] \geq 1 \Rightarrow \left[x + \frac{1}{x}\right] \geq \frac{1}{2} \Rightarrow x + \frac{1}{x} \geq 1$

$\Rightarrow x \geq \frac{1}{x} \Rightarrow Df = \left[\frac{1}{x}, +\infty\right)$

۴

ب) $y = \sqrt{\left[x - \frac{1}{x}\right] + \left[-(x - \frac{1}{x})\right]}$ if $\begin{cases} a \in \mathbb{Z} & [a] + [-a] = 0 \\ a \in \mathbb{Z} & [a] + [-a] = -1 \end{cases} \Rightarrow x - \frac{1}{x} \in \mathbb{Z}$

$\Rightarrow Df = \left\{x \mid x = k + \frac{1}{x} \text{ و } k \in \mathbb{Z}\right\}$

الف) $y = \frac{2}{\sin x - 1} \Rightarrow \sin x - 1 \neq 0 \Rightarrow \sin x \neq 1$

$Df = \mathbb{R} - \left\{2k\pi + \frac{\pi}{2}\right\}$



ب) $y = \frac{2}{2\sin^2 x - 1} \Rightarrow 2\sin^2 x - 1 \neq 0 \Rightarrow 2\sin^2 x \neq 1 \Rightarrow \sin^2 x \neq \frac{1}{2}$

$\Rightarrow \sin x \neq \pm \frac{1}{\sqrt{2}} \Rightarrow \sin x \neq \pm \frac{\sqrt{2}}{2}$

$Df = \mathbb{R} - \left\{k\pi \pm \frac{\pi}{4}\right\}$



۵

الف) $y = \sqrt{y \sin x - 1} \Rightarrow y \sin x - 1 \geq 0 \Rightarrow y \sin x \geq 1 \Rightarrow \sin x \geq \frac{1}{y}$
 $D_f = [2k\pi + \frac{\pi}{y}, 2k\pi + \pi - \frac{\pi}{y}]$



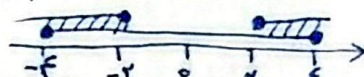
ب) $\sqrt{\cos x - 1} \Rightarrow \cos x - 1 \geq 0 \Rightarrow \cos x \geq 1$

$D_f = \{2k\pi | k \in \mathbb{Z}\}$



الف) $\cos y = |x| - 3 \Rightarrow -1 \leq \cos y \leq 1 \Rightarrow -1 \leq |x| - 3 \leq 1 \Rightarrow -1 + 3 \leq |x| \leq 1 + 3$
 $\Rightarrow 2 \leq |x| \leq 4$
 $|x| \geq 2 \Rightarrow x \geq 2 \vee x \leq -2$
 $|x| \leq 4 \Rightarrow -4 \leq x \leq 4$

$D_f = [-4, -2] \cup [2, 4]$



ب) $y = \arcsin(x^2 + 3x + 1) \Rightarrow -1 \leq x^2 + 3x + 1 \leq 1 \Rightarrow \begin{cases} x^2 + 3x + 1 \geq -1 \rightarrow x^2 + 3x + 2 \geq 0 \rightarrow (x+2)(x+1) \geq 0 \text{ (I)} \\ x^2 + 3x + 1 \leq 1 \rightarrow x^2 + 3x \leq 0 \rightarrow x(x+3) \leq 0 \text{ (II)} \end{cases}$
 الف: $\begin{matrix} \text{(I)} & x & -2 & -1 \\ & + & \phi & -\phi & + \\ \text{(II)} & x & 0 & -3 \end{matrix}$
 $D_f = (I) \cap (II) \Rightarrow [-2, -1] \cup [-1, 0]$

الف) $y = \sqrt{2 - \log_3(x-2)} \Rightarrow 2 - \log_3(x-2) \geq 0 \Rightarrow \log_3(x-2) \leq 2 \Rightarrow x-2 \leq 3^2$
 $\Rightarrow x-2 \leq 9 \Rightarrow x \leq 11, x-2 > 0 \Rightarrow x > 2$ استمرار این دو شرط $D_f = x \in (2, 11)$

ب) $y = \log(x^3 - x) \Rightarrow x^3 - x > 0 \Rightarrow x(x^2 - 1) > 0 \Rightarrow x(x-1)(x+1) > 0$
 $\rightarrow$ سه نقطه: $-1, 0, 1$
 $\begin{matrix} -1 & 0 & 1 \\ -\phi & + & \phi & -\phi & + \end{matrix} \rightarrow D_f = (-1, 0) \cup (1, +\infty)$

الف) $y = \frac{x}{x+1} \Rightarrow x+1 \neq 0 \Rightarrow x \neq -1 \Rightarrow D_f = \mathbb{R}$

ب) $y = \frac{x}{x-1} \Rightarrow x-1 \neq 0 \Rightarrow x \neq 1 \Rightarrow D_f = \mathbb{R} - \{1\}$

ج) $x^2 - 2 \neq 0 \Rightarrow x^2 \neq 2 \Rightarrow (x^2)^{1/2} \neq 2^{1/2} \Rightarrow x^2 \neq 2 \Rightarrow x \neq \pm \sqrt{2}$
 $\Rightarrow D_f = \mathbb{R} - \{\pm \sqrt{2}\}$

د) $y = \frac{x}{x-2} \Rightarrow x-2 \neq 0 \Rightarrow x \neq 2 \Rightarrow x \neq \log_2(2) \Rightarrow D_f = \mathbb{R} - \{\log_2(2)\}$

الف) $y = (x+1)! \Rightarrow x+1 \in \mathbb{N} \Rightarrow x+1 = k \Rightarrow k \in \mathbb{N} \Rightarrow D_f = \{x = \frac{k-1}{1} | k \in \mathbb{N}\}$

ب) $\frac{x^2 - 2}{x - 2} = k \rightarrow k \in \mathbb{N} \quad x - 2 \neq 0 \Rightarrow x \neq 2 \Rightarrow x \neq 2/1$

$x^2 - 2 = k(x - 2) \Rightarrow x^2 - 2 = kx - 2k \Rightarrow x^2 - kx = 2 - 2k$

$\Rightarrow x(x - k) = 2 - 2k \Rightarrow x = \frac{2 - 2k}{x - k} \rightarrow$ (توجه: x و y متغیری از مجموعه اعداد حقیقی هستند و k از مجموعه اعداد صحیح است)

$D_f = \left\{ \frac{2 - 2k}{x - k} | k \in \mathbb{N} \right\}$

(توجه: x و y متغیری از مجموعه اعداد حقیقی هستند و k از مجموعه اعداد صحیح است)