

$$y = \sqrt{x f(x)} \Rightarrow \begin{array}{c} -a \quad 0 \quad 2 \quad a \\ - \quad + \quad - \quad + \quad - \\ \circ \quad \circ \quad \circ \quad \circ \quad \circ \end{array}$$

$$x f(x) \geq 0$$

$$D_f = [-a, 0] \cup [2, a]$$

$$\frac{-x}{f(x)} \geq 0 \Rightarrow f(x) \leq 0 \Rightarrow \begin{array}{c} -a \quad -2 \quad 0 \quad 2 \quad a \\ - \quad + \quad - \quad + \quad - \\ \circ \quad \circ \quad \circ \quad \circ \quad \circ \end{array}$$

$$y = \sqrt{-x} \Rightarrow \begin{array}{c} 0 \quad a \\ \circ \quad \circ \\ - \quad + \quad - \quad + \quad - \\ \circ \quad \circ \quad \circ \quad \circ \quad \circ \end{array}$$

$$D_f = (-2, 0) \cup (0, 2)$$

$$f(f(a) + f(f(1))) \geq ? \Rightarrow f(x) = \begin{cases} x - \sqrt{x+4} & x > 3 \\ x + 3 & x \leq 3 \end{cases}$$

$$\textcircled{1} f(1) \leq x + 3 \Rightarrow a \rightarrow f(a) = x - \sqrt{x+4} \rightarrow a - 3 = 2$$

$$f(f(a) + 1) \Rightarrow f(a) \geq 2 \rightarrow f(2) = 2x + 3 \Rightarrow 2$$

$$\Rightarrow 2 + 2 = 4$$

$$\textcircled{2} \checkmark$$

$$f(x) - 2f(2) = x^2 - 4x + 4$$

$$\hookrightarrow x = 2 \Rightarrow f(2) - 2f(2) = (2)^2 - 4(2) + 4 \Rightarrow -f(2) = 0 \Rightarrow f(2) = 0$$

$$f(2) = 0 \Rightarrow f(x) - 2(0) = x^2 - 4x + 4 \Rightarrow f(x) + 4 = x^2 - 4x + 4$$

$$\Rightarrow f(x) = x^2 - 4x$$

$$f(-2) = (-2)^2 - 4(-2) = 4 + 8 = 12 \rightarrow f(-2) = 12$$

$$f(x) = ax^2 - bx + 1 \xrightarrow{x=1} f(1) = a - b + 1 \rightarrow a - b \geq f(1) - 1, f(0) \geq 1$$

$$f(x-1) - f(x) \geq 9x + 1 \xrightarrow{x=1} f(0) - f(1) \geq 9 + 1 \rightarrow f(1) \geq 2 - 1 = 1$$

$$a - b \geq -9 - 1 = -10$$

$$\textcircled{3} \checkmark$$

$$f(x) = (x)^2 - x(x) + 1 = 9 - 9 + 1 = 1$$

$$f(-1) = (-1)^2 - x(-1) + 1 = 1 + 9 + 1 = 11$$

$$f(1) = (1)^2 - x(1) + 1 = 1 - 9 + 1 = -7 \quad \text{✓}$$

$$\frac{x f(x) + f(-1)}{(f(1))^2} = \frac{x(1) + 11}{(-7)^2} = \frac{x + 11}{49} = 1$$

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$$f(x) = x(x) + 1 = 12 + 1 = 13 \rightarrow \text{اگر } x \geq 2 \text{ اول}$$

$$x < 2 \rightarrow f(-x) = (-x)^2 = x^2$$

$$x \geq 2 \rightarrow f(x) = x(x) + 1 = 4 + 1 = 5 \quad \text{✓}$$

$$\frac{x f(x) - x f(-x)}{f(x)} = \frac{x(13) - x(4)}{5} = \frac{9x}{5} = \frac{9}{5}$$

صورتی که در صورت

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$$f(x) = 1 \rightarrow f(f(x)) = f(1) = 4$$

$$f(x) = 2 \rightarrow f(f(x)) = f(2) = 4$$

$$f(1) = 4 \rightarrow \text{مستقیم}$$

$$\frac{f(f(x)) - f(f(2))}{f(1) - 2} = \frac{4 - 4}{4 - 2} = \frac{0}{2} = 0$$

مستقیم

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$$f(n) = \frac{n^2 + 4n + 4}{n^2 + 4n + 1 + 4} \rightarrow f(n) = \frac{(n+2)^2 + 4}{(n+2)^2 + 1}$$

$$n = \sqrt{x} - 2 \rightarrow n + 2 = \sqrt{x} \rightarrow (n+2)^2 = x \rightarrow f(\sqrt{x} - 2) = \frac{x + 4}{x + 1} = \frac{1}{x} = 1$$

$$f(x) = \frac{x(x) + f(2) + 4}{(x)^2 + f(2) + 4} = \frac{4 + 1 + 4}{4 + 1 + 4} = \frac{9}{9} = 1 \quad \text{✓}$$

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$$f(\sqrt{n}) = (\sqrt{n})^2 + \frac{1}{(\sqrt{n})^2} = n + \frac{1}{n}$$

$$(f(\sqrt{n}))^2 = (n + \frac{1}{n})^2 = n^2 + 2(n)(\frac{1}{n}) + \frac{1}{n^2} = n^2 + \frac{1}{n^2} + 2$$

$$(f(\sqrt{n}))^2 - f(n) = (n^2 + \frac{1}{n^2} + 2) - (n^2 + \frac{1}{n^2}) = 2$$

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