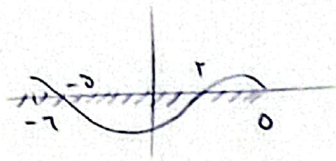


$$y = \sqrt{xf(x)} \Rightarrow xf(x) \geq 0$$

$$D_f = [-5, 0] \cup [2, 5]$$



۱

$$y = \sqrt{\frac{-x}{f(x)}} \Rightarrow f(x) \neq 0 \quad \frac{-x}{f(x)} \geq 0$$

$$\frac{-x}{f(x)} \geq 0 \Rightarrow [-5, 0] \cup [2, 5]$$

$$-5, -2, -1, 2, 3, 4, 5 \Rightarrow \text{همه صحیح}$$

۲

$$f(x) - 2f(x) = x^2 - 5x + 4$$

$$f(2) = f(2) - 2f(2) = 4 - 10 + 4 \Rightarrow f(2) = -2$$

$$f(-2) = f(-2) - 2f(-2) = 4 + 10 + 4 \Rightarrow f(-2) = 1$$

۳

$$f(x) = \begin{cases} x - \sqrt{x+4} & x > 4 \\ 2x + 3 & x \leq 4 \end{cases}$$

$$f(1) = 5$$

$$f(5) = 5 - 3 = 2$$

$$f(5) = 2$$

$$f(2) = 4 + 3 = 7$$

$$f(f(5)) + f(f(1)) = ?$$

$$2 + 7 = 9$$

۴

$$f(x) = ax^2 - bx + 2 \quad f(x-1) - f(x) = 7x + 2$$

$$\Rightarrow f(x-1) = a(x-1)^2 - b(x-1) + 2 - ax^2 + bx - 2 = 7x + 2$$

$$\Rightarrow x=1 \Rightarrow a(x-1)^2 - b(x-1) - ax^2 + bx = 7x + 2$$

$$-a + b = 1 \Rightarrow a - b = 1$$

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$$f(n) = n^2 - 2n + 1$$

$$\frac{r f(2) + f(-1)}{f'(r)} = \frac{r(4 - 4 + 1) + 1 + 2 + 1}{(2 - 2 + 1)^2} = \frac{r + 4}{1} = r$$

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$$f(n) = \begin{cases} r n + 1 & n \geq r \\ n^2 & n < r \end{cases}$$

$$\frac{r f(2) - r f(1-r)}{f'(r)} = \frac{r(1+r) - r(2)}{r} = \frac{r}{r}$$

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$$f = \{(r, 2), (2, r), (2, 1), (1, 2)\}$$

$$\frac{\cancel{f(2)} - \cancel{f(1-r)}}{\cancel{f(1-r)}} = \frac{4 - 4}{2} = 0$$

8

$$f(n) = \frac{n^2 + 2n + 9}{n^2 + 2n + 4}$$

$$f(\sqrt{r}-1) = \frac{(\sqrt{r}-1)^2 + 2(\sqrt{r}-1) + 9}{(\sqrt{r}-1)^2 + 2(\sqrt{r}-1) + 4} = \frac{r + 2 - 2\sqrt{r} + 2\sqrt{r} - 2 + 9}{r + 2 - 2\sqrt{r} + 2\sqrt{r} - 2 + 4} = \frac{r + 9}{r + 4} = r$$

$$f(r) = \frac{r^2 + 2r + 9}{r^2 + 2r + 4} = \frac{r}{r}$$

$$f(n) = n^2 + \frac{1}{n^2}$$

$$f(\sqrt{n}) = (\sqrt{n})^2 + \frac{1}{(\sqrt{n})^2} = n + \frac{1}{n} \Rightarrow (f(\sqrt{n}))^2 = \left(n + \frac{1}{n}\right)^2$$

$$(f(\sqrt{n}))^2 - f(n) = n^2 + \frac{1}{n^2} + 2 - n^2 - \frac{1}{n^2} = 2$$

$$\downarrow$$

$$n^2 + \frac{1}{n^2} + 2$$

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