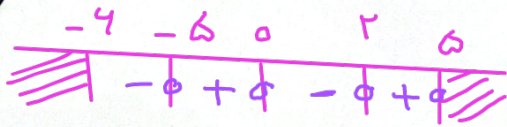
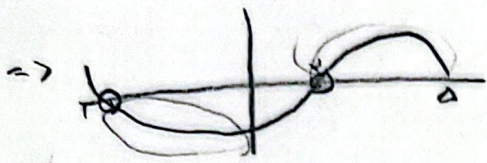


$$a(f)u \geq 0 \Rightarrow f(u) \times u \geq 0$$

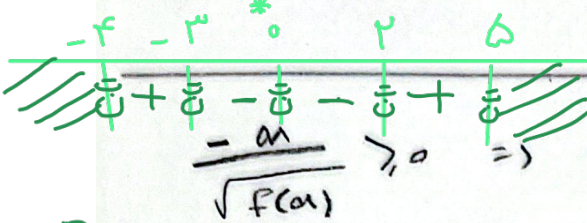


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$$\Rightarrow [-5, 0] \cup [2, 5]$$

حالت اول: $f(u) < 0, u > 0$



حالت دوم: $f(u) > 0, u < 0$

$$\frac{-u}{\sqrt{f(u)}} \geq 0 \Rightarrow -u \geq 0 \Rightarrow u \leq 0$$

$$f(u) \neq 0$$

$$D_f = (-3, 2) - \{0\}$$

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$$f(u) > 0 \Rightarrow -3 < u < 0 \Rightarrow -2, -1 \Rightarrow \text{خطی}$$

$$f(2) = f(2) - f(2) = 5 - 9 + 4 \Rightarrow -f(2) = 2 \Rightarrow -2f(2) = 4$$

$$f(-2) \Rightarrow f(-2) + 4 = 5 + 9 + 4 \Rightarrow f(-2) + 4 = 18 \Rightarrow f(-2) = 14$$

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$$f(0) = u - \sqrt{u+4} \Rightarrow 0 - \sqrt{4} = 0 - 2 = -2$$

$$f(2) = 2u + 2 \Rightarrow 5 + 2 = 7 \text{ ①}$$

$$f(1) = 2 + 2 = 4$$

$$f(0) = 0 - \sqrt{4} = 0 - 2 = -2 \text{ ②}$$

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$$f(u) = au^2 - bu + 4$$

$$f(u+1) = a(u+1)^2 - b(u+1) + 4 \Rightarrow a(u^2 + 2u + 1) - bu - b + 4$$

$$= au^2 + 2au + a - bu - b + 4$$

۲

$$(au^2 + 2au + a - bu - b + 4) - (au^2 - bu + 4) = -2au + a - b$$

$$-2au + a - b = 4a + 4 \Rightarrow -2a = 4 \Rightarrow a = -2 \Rightarrow b = 4 \Rightarrow a - b = -6$$

$$f(x) = x^2 - 2(x) + 1 = x + 1$$

$$f(-1) \Rightarrow f(-1) = (-1)^2 - 2(-1) + 1 \Rightarrow 1 + 2 + 1 = 4$$

1, 2, 3

$$f(x) + f(-1) = f(1) + 4 \Rightarrow x + 4 = 5 \quad \checkmark$$

$$\frac{x}{1} = x$$

$$f^x(x) = (f(x))^x = (-1)^x = 1 \Rightarrow \boxed{x=1} \quad \checkmark$$

$$\frac{f(x)(1+x) - xf(x)}{x+1} = \frac{x^2 - 1}{x+1} = \frac{(x-1)(x+1)}{x+1} = x-1 \quad \checkmark$$

1, 2

$$\left. \begin{aligned} f(1) &= 1 \Rightarrow f(1) = 4 \\ f(2) &= 2 \Rightarrow f(2) = 9 \\ f(1) &= 4 - 2 = 2 \end{aligned} \right\}$$

$$\frac{4-2}{2} = 1$$

2 $\checkmark$

9. از اتحاد مربع دو جمله‌ای استفاده می‌کنیم:

$$f(n) = \frac{n^2 + 4n + 4}{n^2 + 4n + 4} = \frac{n^2 + 4n + 4 + 0}{n^2 + 4n + 4 + 1} = \frac{(n+2)^2 + 0}{(n+2)^2 + 1}$$

$$f(\sqrt{x}-2) = \frac{(\sqrt{x}-2+2)^2 + 0}{(\sqrt{x}-2+2)^2 + 1} = \frac{x + 0}{x + 1} = 1 \quad \checkmark$$

$$f(f(\sqrt{x}-2)) = f(1) = \frac{1^2 + 0}{1^2 + 1} = \frac{1}{2}$$

$$f(\sqrt{m}) = (\sqrt{m})^2 - \frac{1}{(\sqrt{m})^2} = m - \frac{1}{m} \Rightarrow f(\sqrt{m})^2 = \left(m - \frac{1}{m}\right)^2$$

$$m^2 - 2 + \frac{1}{m^2} = m^2 - 2 + \frac{1}{m^2} \quad \checkmark$$

$$\Rightarrow f(m) = m^2 + \frac{1}{m^2} \Rightarrow (f(\sqrt{m})^2) - f(m) = m^2 - 2 + \frac{1}{m^2} - (m^2 + \frac{1}{m^2}) = -2$$

2