

$$y = \sqrt{x f(x)}$$

$$x f(x) > 0$$

↓ ↓  
0 0

|          | $(-\infty, -1)$ | $-1$ | $(-1, 0)$ | $0$ | $(0, 1)$ | $1$ | $(1, \infty)$ | $\infty$ |
|----------|-----------------|------|-----------|-----|----------|-----|---------------|----------|
| $x$      | -               | -    | -         | 0   | +        | +   | +             | +        |
| $f(x)$   | +               | 0    | -         | -   | -        | 0   | +             | 0        |
| $x f(x)$ | -               | 0    | +         | 0   | -        | 0   | +             | 0        |

$$D_f = [-\infty, 0] \cup [1, \infty)$$

$$\frac{-x}{f(x)} > 0$$

$$-x, 0, 1, \infty$$

| $x$               | $(-\infty, 0)$ | $(0, 1)$ | $(1, \infty)$ |
|-------------------|----------------|----------|---------------|
| $\frac{-x}{f(x)}$ | +              | +        | -             |

$$D_f = (-\infty, 0) \cup (0, 1) \rightsquigarrow \lim_{x \rightarrow 0} \frac{-x}{f(x)} = 1$$

$$f(x) = x f(x) = x^2 - 3x + 1$$

$$\begin{aligned} f(1) - x f(x) &= 1 - 1 + \varepsilon \rightarrow f(-1) - x f(x) = 1 + 1 + \varepsilon \\ -f(1) &= 1 \\ f(1) &= -1 \end{aligned}$$

$$f(-1) = 1$$

$$f(f(a)) + f(f(1))$$

$$f(a) = a - \sqrt{a} = 1$$

$$1 + 1 = 2$$

$$f(f(a)) = 1 \times 1 + 1 = 2$$

$$f(1) = 1 \times 1 + 1 = 2 \rightarrow f(f(1)) = 1$$

$$f(x) = ax^2 - bx + 1$$

$$f(n-1) = a(n-1)^2 - b(n-1) + 1 = a(n^2 + 1 - 2n) - bn + b + 1$$

$$an^2 + a - 2an - bn + b + 1 = an^2 + bn + 1 = 4n + 1$$

$$-2an + a + b = 4n + 1$$

$$-2an = 4n$$

$$a = -2$$

$$a + b = 1$$

$$-2 + b = 1$$

$$b = 3$$

$$a - b$$

$$-2 - 3 = -5$$

$$f(3) = 9 - 9 + 1 = 1$$

$$f(-1) = 1 + 3 + 1 = 5$$

$$f(2) = 8 - 4 + 1 = 5$$

$$\frac{f(3) + f(-1)}{f(2)} = \frac{1 + 5}{5} = \frac{6}{5} = \boxed{\frac{6}{5}}$$

$$f(5) = 25 - 5 + 1 = 21$$

$$f(-2) = 4$$

$$f(2) = 5$$

$$\frac{f(5) - f(-2)}{f(2)} = \frac{21 - 4}{5} = \frac{17}{5} = \boxed{\frac{17}{5}}$$

$$f = \{(1, 9), (9, 1), (4, 4)\}$$

$$\frac{f(f(1)) - f(f(9))}{f(1) - 9} = \boxed{0}$$

$$f(x) = \frac{x^2 + 5x + 9 - 2 + 2}{x^2 + 5x + 2 + 1 - 1} = \frac{\sqrt{x(x+1)} + 2}{(x+1)^2 + 1} = \frac{x+2}{x+1} = \frac{1}{x} = x$$

$$x = \sqrt{x} - 1 \rightarrow x + 1 = \sqrt{x}$$

$$f(f(x)) \Rightarrow f(1) = \frac{(1+1)^2 + 2}{(1+1)^2 + 1} = \frac{5}{5} = \boxed{1}$$

$$f(x) = x^2 + \frac{1}{x^2}$$

$$f(\sqrt{x}) = x + \frac{1}{x} \rightarrow f(\sqrt{x})^2 = x^2 + \frac{1}{x^2} + 2$$

$$(f(\sqrt{x}))^2 - f(x) = (x^2 + \frac{1}{x^2} + 2) - (x^2 + \frac{1}{x^2}) = 2 = \boxed{2}$$