

باید x و $f(x)$ هم علامت باشند یا حداقل یکی از آنها صفر باشد تا عبارت تعریف شود.

$$D_f = [-5, 0] \cup [2, 5]$$

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x و $f(x)$ نباید هم علامت و صفر باشند.

$$D_f = (-3, 0) \cup (0, 2)$$

اعداد صحیح: $\{-2, -1, 1\}$

۳ تا عدد صحیح

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$$\begin{aligned} f(n) - 2f(2) &= x^2 - 3n + 4 \Rightarrow f(-2) = ? \Rightarrow f(-2) - 2f(2) = (-2)^2 - 3(-2) + 4 \Rightarrow f(-2) - 2f(2) = 14 \\ f(2) &\Rightarrow f(2) - 2f(2) = 4 - 6 + 4 = 2 \Rightarrow -f(2) = 2 \\ \left\{ \begin{array}{l} f(-2) - 2f(2) = 14 \\ -f(2) = 2 \end{array} \right. &\Rightarrow \begin{array}{l} f(-2) - 2f(2) = 14 \\ -2f(2) = 4 \end{array} \\ &\Rightarrow \boxed{f(-2) = 10} \end{aligned}$$

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$$f(x) = \begin{cases} x - \sqrt{x+4} & [4, +\infty) \\ 2x + 3 & (-\infty, 3) \end{cases} \quad f(f(5)) + f(f(1)) = 7 + 2 = 9$$

$$\begin{cases} f(5) = 5 - \sqrt{5+4} = 2 \\ f(1) = 2(1) + 3 = 5 \\ f(2) = 2(2) + 3 = 7 \end{cases}$$

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$$f(n) = an^2 - bn + 2 \quad f(n-1) = a(n-1)^2 - b(n-1) + 2 = a(n^2 + 1 - 2n) - bn + b + 2 = an^2 + a - 2an - bn + b + 2$$

$$\Rightarrow f(n-1) - f(n) = an^2 + a - 2an - bn + b + 2 - an^2 + bn - 2 = a - 2an + b = 4n + 2$$

$$\begin{aligned} -2an + a + b &= 4n + 2 \Rightarrow -2an = 4n \Rightarrow a = -2 \\ a + b &= 2 \Rightarrow -2 + b = 2 \Rightarrow b = 4 \end{aligned} \quad \left. \begin{array}{l} a = -2 \\ b = 4 \end{array} \right\} a - b = -2 - 4 = -6$$

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$$f(x) = x^2 - x(-1) + 1 = 1 \quad f(-1) = (-1)^2 - x(-1) + 1 = 0 \quad f(x) = x^2 - x + 1 = -1$$

$$\frac{x f(x) + f(-1)}{f'(x)} = \frac{x + 0}{1} = x$$

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$$f(x) = \begin{cases} x^2 + 1 & [2, +\infty) \\ x^2 & (-\infty, 1] \end{cases} \Rightarrow f(x) = x^2 + 1 = 13, f(-2) = (-2)^2 = 4$$

$$\frac{x f(x) - x f(-2)}{f'(x)} = \frac{x(13) - x(4)}{2x} = \frac{9x}{2x} = \frac{9}{2}$$

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$$\frac{f(f(x)) - f(f(x))}{f(1) - x} = \frac{x - x}{x - 1} = 0$$

$$\begin{cases} f(x) = 1 \\ f(x) = x \\ f(1) = x \\ f(x) = x \end{cases}$$

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$$f(\sqrt{x} - 2) = ? \Rightarrow f(\sqrt{x} - 2) = \frac{(\sqrt{x} - 2)^2 + f(\sqrt{x} - 2) + 9}{(\sqrt{x} - 2)^2 + f(\sqrt{x} - 2) + 0} = \frac{x + 4 - 4\sqrt{x} + \sqrt{x} - 1 + 9}{x + 4 - 4\sqrt{x} + \sqrt{x} - 1 + 0} = \frac{1}{x} = x$$

$$f(x) = ? \Rightarrow f(x) = \frac{x^2 + f(x) + 9}{x^2 + f(x) + 0} = \frac{1}{1} \Rightarrow f(x) = \frac{1}{1}$$

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$$\left. \begin{aligned} f(x) &= x^2 + \frac{1}{x^2} \\ f(\sqrt{x}) &= (\sqrt{x})^2 + \frac{1}{(\sqrt{x})^2} = x + \frac{1}{x} \end{aligned} \right\} (f(\sqrt{x}))^2 - f(x) = (x + \frac{1}{x})^2 - x - \frac{1}{x} = x^2 + \frac{1}{x^2} + 2 - x - \frac{1}{x} = x^2 + \frac{1}{x^2} = x$$

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