پاسخنامه تکلیف شماره ۱۳ : [illegible] دهم A

۱) $a_1 = 40 \quad a_4 = 61 \rightarrow a_4 = a_1 + 3d$

$61 = 40 + 3d \Rightarrow 3d = 21 \Rightarrow \boxed{d = 7} \rightarrow a_n = 7n + 33 = 208$

$7n = 175 \rightarrow n = \boxed{25}$

۲) $101 , 108 , 115 , \ldots , 997 \xrightarrow{\text{جـ عمومی}} 7n + 94 < 1000 \rightarrow 7n < 906$

$n < 129$

$7(129) + 94 = 997 \longrightarrow \frac{997 - 101}{7} + 1 = \boxed{129}$

۳) $a_{n+1} + a_{n+2} + a_{n+3} = -6n + 12 \rightarrow a_1 + nd + a_1 + (n+1)d + a_1 + (n+2)d$

$3a_1 + 3nd + 3d = -6n + 12 \rightarrow 3nd = -6n \rightarrow d = -2 \Rightarrow a_1 = 6$

$a_{17} + a_8 = a_1 + 16d + a_1 + 7d = 2a_1 + 23d \Rightarrow 2(6) + 23(-2) = \boxed{-34}$

۴) $a_{13} - a_8 = a_8 - a_3 \qquad 2a_8 = a_{13} + a_3 \qquad 2(2^{n+1}) = (2^{2n} - 3) + (2^n - 1)$

$2^{n+2} = 2^{2n} - 3 + 2^n - 1 \xrightarrow{2^n = t} 4t = \underbrace{t^2 - 3 + t - 1}_{3t-4} \Rightarrow t = 4 \rightarrow 2^n = 4 \rightarrow \boxed{n = 2}$

$a_3 = 2^2 - 1 = 3 \qquad a_8 = 2^{2+1} = 8 \qquad a_{13} = 2^4 - 3 = 13 \qquad 3 + 8 + 13 = \boxed{24}$

۵) $t_{12} - t_{10} = 5$

$t_{12} + t_{10} = 25$

$2t_{12} = 30 \rightarrow t_{12} = 15 , t_{10} = 10 \rightarrow d = \frac{15 - 10}{12 - 10} = 2/5$

$a_{21} = a_{12} + 9d = 15 + \frac{45}{2} = 37/5$

۱۶) جمع پنج جمله اول $S_5 = \frac{5}{2}\left(a_1 + \overbrace{(a_1+4d)}^{a_5}\right) = \frac{5}{2}(2a_1+4d)$

جمع پنج جمله دوم $\Rightarrow a_6+a_7+a_8+a_9+a_{10} \Rightarrow a_1+5d+a_1+6d+a_1+7d+a_1+8d+a_1+9d = 5a_1+35d$

$\frac{5}{2}(2a_1+4d) = \frac{1}{3}(5a_1+35d) \rightarrow 15(2a_1+4d) = 2(5a_1+35d)$

$30a_1+60d = 10a_1+70d \rightarrow d = 2a_1 \rightarrow a_1, 3a_1, 5a_1, 7a_1, \dots$

a=۱ باشد ۳، ۵، ۷، ... نسبت جمله دوم به اول $= \frac{3}{1} = \boxed{3}$

۱۷) $a_n = a_{n-3}+15 \rightarrow a_1+((n-3)-1)d+15 = a+(n-4)+15 \rightarrow d=5$

$a_9 = a_1+8d \rightarrow a_1+8(5) = a_1+40$ $\quad a_5 = a_1+4d \rightarrow a_1+4(5) = a_1+20$

$a_7 = a_1+6d \rightarrow a+6(5) = a_1+30$ $\quad a_9^2 - a_5^2 = (a_9+a_5)(a_9-a_5)$

$\left(\underbrace{(a+40)+(a+20)}_{2a+60}\right) \times \left(\underbrace{(a+40)-(a+20)}\right) = (2a+60)(20) = 40a+1200$

$\frac{40a+1200}{a+30} = \boxed{40}$ برابر

۱۸)

$d = \frac{2+\sqrt{3}-2(1-6\sqrt{3})}{13} = \frac{2+\sqrt{3}-2+12\sqrt{3}}{13} = \frac{13\sqrt{3}}{13} = \sqrt{3}$

۱۹) $a_1 = 2, \; a_n = 32 \rightarrow n-2 = 4n-3 \rightarrow n = 4n-1$

$11 = 2+nd \rightarrow nd = 9 \rightarrow d = \frac{9}{n} \rightarrow a_{4n-1} = a_1+((4n-1)-1)d$

$32 = 2+(4n-2)d$

$30 = (4n-2)d \rightarrow 15 = (2n-1)d$

$15 = (2n-1)\times\frac{9}{n}$

$15n = 9(2n-1) \rightarrow 15n = 18n - 9 \rightarrow n = 3$

15)

$a_n + a_{n+4} = \underline{a_3 + a_{17}}$

$(a_1 + 2d + a_1 + 16d) = 2a_1 + 18d \rightarrow 2n + 2 = 18 \rightarrow n = 8$

$a_n = 3a_{\frac{n}{2}} \rightarrow 3a_4 = a_8$

$a_1 + 7d = 3a_1 + 9d \rightarrow a_1 = -d \rightarrow a_m = (a_1 + (n-1)d) \Rightarrow -d + (m-1)d = 0$

$\Rightarrow d(m-2) = 0$

اگر d صفر باشد دنباله ثابت می‌شود پس قطعا $m-2$ برابر صفر است $\Leftarrow$ $m-2=0 \Rightarrow \underline{m=2}$

$\Rightarrow \underline{a_2 = 0}$