سارا عزیزی کلاس ۱۳۰

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(۱)
$a_1=40, a_4=61 \Rightarrow d=\frac{61-40}{4-1}=7 \Rightarrow a_n=a+7(n-1)=208 \Rightarrow 7+7n-7=208$

$\Rightarrow 7n=175 \Rightarrow \boxed{n=25}$

۲ ✓

(۲)
$7\times14+3=101 \Rightarrow 101, 108, \dots, 997 \Rightarrow \frac{997-101}{7}+1=\boxed{129}$

۲ ✓

(۳)
$a_{n+1}+a_{n+2}+a_{n+3}=-6n+12 \Rightarrow 3a+3(n+1)d=-6n+12 \Rightarrow a+(n+1)d=-2n+4$

$\Rightarrow a+nd+d=-2n+4 \Rightarrow nd=-2n \Rightarrow d=-2, a+d=4 \Rightarrow a-2=4 \Rightarrow a=6$

$\Rightarrow a_{17}+a_{7}=2a+22d=2\times6+22\times(-2)=\boxed{-32}$

۲ ✓

(۴)
$a_2=2^n-1$ $\quad 2^n-1+5d=2^{n+1} \Rightarrow 5d=2^n(2^1-1)$

$a_7=2^{n+1}=a_2+5d$ $\quad \Rightarrow 5d=2^n+1$

$a_{12}=2^{2n}-3=a_7+5d$ $\quad 2^{2n}-3=2^{n+1}+2^n+1 \Rightarrow 2^{2n}=2^{n+1}+2^n+4$

$\Rightarrow 2^n(2^n-2-1)=4 \Rightarrow 2^n=y \Rightarrow y(y-3)=4$

$\Rightarrow y^2-3y-4=0 \Rightarrow (y-4)(y+1)=0 \Rightarrow y=-1=2^n$ ✗

$y=4=2^n$ ✓ $\Rightarrow n=2 \Rightarrow 3, 8, 13 \Rightarrow 3+8+13=\boxed{24}$

۲ ✓

(۵)
$a_{10}-a_{12}=a+9d-a-11d=5 \Rightarrow d=-2.5$

$a_{10}+a_{12}=a+9d+a+11d=2a+20d=2a-50=25 \Rightarrow 2a=75 \Rightarrow a=37.5$

$a_{21}=a+20d=37.5+(20\times-2.5)=\boxed{12.5}$

۲ ✓

۹) $a_1+a_2+a_3+a_4+a_5=\frac{1}{2}(a_4+a_6+a_8+a_{10})$

$a_{n+10}d=\frac{1}{2}(a_n+2a_n d)=\frac{a}{2}a+\frac{2a}{2}d \to \frac{10}{2}a=\frac{a}{2}d \to a=\frac{1}{2}d$

$\to \frac{a_2}{a}=\frac{a+d}{a}=\frac{\frac{1}{2}d+d}{\frac{1}{2}d}=\frac{\frac{3}{2}d}{\frac{1}{2}d}=3$ ✓

۷) $t_n=t_{n-2}+10=t_n+2d+10 \to 2d=10 \to d=5$

$\frac{t_9^2-t_5^2}{t_7}=\frac{(t_9-t_5)(t_9+t_5)}{t_7}=\frac{8d(2+7)}{6\times 7}=20\times 2=40$ ✓

۸) $\frac{2+\sqrt{2}-4+12\sqrt{2}}{12}=\frac{12\sqrt{2}}{12}=\sqrt{2}$ ✓

۹) $\frac{20-2}{2n-2}=d \quad a_{n+1}=11 \to a+nd=11 \to a_1+\frac{20n}{2n-2}=11 \to \frac{18n}{2n-1}=9$

$\to 18n=18n-9 \to n=2$ ✓

۱۰) $a_n+a_{n+4}=a_2+a_{17} \to 2n+1=2n+4 \to n=4$

$a_n=2d\frac{n}{2} \to a_1=2a_4 \to a+7d=2a+9d \to 2a+2d=0 \to a+d=0=a_2$ ✓