حلیا عرب

۲۰

سوال ۱)

$a_1 = 40 \quad a_{41} = 61 \qquad d = \frac{61-40}{41-1} = \frac{21}{40} = ⑦ = d$

$7n + b = 40$

$b = 33 \qquad 7n + 33 = 208$

$7n = 175$

$n = 25$

← ۲۵امین جمله

۲ ✓

سوال ۲)

$101, \ldots, 997$

$a_1 \qquad a_n$

$\frac{997-101}{7} + 1 = 129$

۲ ✓

سوال ۳)

$a_{17} + a_8 = a_{13} + a_{12} = a_{11} + a_{14}$

$n = 10 \Rightarrow a_{11} + a_{12} + a_{13} = -48$

$n = 11 \Rightarrow a_{12} + a_{13} + a_{14} = -54$

$\Rightarrow \underbrace{a_{13} + a_{12}}_{a_{17}+a_8} + \underbrace{a_{13} + a_{12}}_{a_{17}+a_8} + \underbrace{a_{11} + a_{14}}_{a_{17}+a_8} = -102$

$3(a_{17} + a_8) = -102 \Rightarrow a_{17} + a_8 = -34$

۲ ✓

سوال ۴)

$2^{2n} - 3 - 2^{n+1} = 2^{n+1} - 2^n - 1 \rightarrow 2^{2n} - 3 - (2^n \times 2^1) - 2^n - 1 = 0$

$2^{2n} - 3 - 2^n(2+1) - 1 = 0$

$2^{2n} - 3 \times 2^n - 4 = 0$

$\Delta = b^2 - 4ac \Rightarrow 9 - 4(-4) = 25 \,/\, n = \frac{-b \pm \sqrt{\Delta}}{2a}$

$\frac{3 \pm \sqrt{25}}{2}$

$2^n = 4 \Rightarrow n = 2$

$2^n = -1$ ✗ نمی‌شود

$a_4 = 2^2 - 1 \rightarrow 4 - 1 = 3$

$a_8 = 2^{2+1} = 2^3 = 8$

$a_{13} = 2^4 - 3 = 13$

$13 + 8 + 3 = 24$

۲ ✓

سوال ۵)

$a_{12} - a_{10} = 5 \Rightarrow a_{12} = 15$

$a_{12} - a_{10} = 5$

$a_{12} + a_{10} = 25$

$-2a_{10} = 5 - 25 \Rightarrow a_{10} = 10$

$a_{10} = 10, \; a_{12} = 15 \quad \frac{15-10}{2} \Rightarrow d = 2.5$

$a_{21} = a_{12} + 9d \Rightarrow 15 + 9(2.5) = 37.5$

۲ ✓

سوال ۶)

$a_1 + a_2 + a_3 + a_4 + a_5 \Rightarrow \frac{5}{2}(2a_1 + 4d) \qquad \frac{1}{4}(a_6 + a_7 + a_8 + a_9 + a_{10})$

$\frac{5}{2}(2a_1 + 4d) = \frac{1}{4} \times \frac{5}{2}(2a_1 + 14d) \qquad \frac{5}{2}(2a_6 + 4d) \Rightarrow \frac{5}{2}(2a_1 + 14$

$2a_1 + 4d = \frac{2}{4}a_1 + \frac{14}{4}d \Rightarrow 6a_1 = 2d \Rightarrow a_1 = \frac{1}{3}d$

$\frac{a_2}{a_1} = \frac{a_1 + d}{a_1} = \frac{\frac{1}{3}d + d}{\frac{1}{3}d} = \frac{4}{1} = ③$

۲ ✓

سوال ۷) $t_n = t_{n-2} + 10 \Rightarrow t_n - t_{n-2} = 10 \rightarrow n=4 \rightarrow t_4 - t_2 = 2d$

$d = 5$

$t_9 = n=9 \rightarrow t_9 + 10$

$t_5 \rightarrow n=5 \rightarrow t_7 + 10$

$t_9^2 - t_5^2 \Rightarrow (t_9 - t_5)(t_9 + t_5) \rightarrow (t_7 + 10 - t_7 - 10)(t_7 + 10 + t_7 + 10)$

$4d = 20$

$2t_1 + 6d + 30 \rightarrow 2t_1 + 120$

$20 \times (2t_1 + 60) = t_7$

$40t_1 + 1200 \longrightarrow \frac{40t_1 + 1200}{t_1 + 30} = 40$ ✓ ۲

سوال ۸) $a_1 = 2(1 - 6\sqrt{3})$ $\quad d = \frac{2 + \sqrt{3} - 2(1 - 6\sqrt{3})}{12 + 1} = 2 + \sqrt{3} - 2 + 12\sqrt{3} =$

$a_{13} = 2 + \sqrt{3}$ ✓ ۲ ← تعداد رابطه‌ها

$\frac{13\sqrt{3}}{13} = \sqrt{3} = d$

سوال ۹) $d = \frac{32 - 2}{4n - 3 + 1} \Rightarrow \frac{15}{2n - 1}$ واسطه‌ی ۱۱

$a_{n+1} = 11 \rightarrow a_1 + nd \Rightarrow 2 + nd = 11$

$nd = 9$

$n\left(\frac{15}{2n-1}\right) = 9$

$n = 3$

✓ ۲

سوال ۱۰) $a_n + a_{n+4} = a_4 + a_{12}$

$n + n + 4 = 20 \Rightarrow 2n = 16 \Rightarrow n = 8$

$\frac{a_n}{a_{\frac{n}{2}}} = 3$ $\quad \frac{a_8}{a_4} = 3 \Rightarrow \frac{a_1 + 7d}{a_1 + 3d} = 3 \rightarrow a_1 + 7d = 3a_1 + 9d$

$-2d = 2a_1$

$-d = a_1$

✓ ۲

$a_1 + (n-1)d = 0 \rightarrow a_1 + (n-1) - a_1 = 0$

$a_1 - na_1 + a_1 = 0 \Rightarrow 2a_1 - na_1 = 0$

$a_1(2 - n) = 0$

$n = 2 \rightarrow$ جمله دوم

$\hookrightarrow a_1 = 0 \rightarrow d = 0$ ✗