

$$a_1 = 40$$

$$a_4 = a + 3d = 91$$

$$a_n = 208$$

$$a_4 = 40 + 3d = 91 \rightarrow 3d = 51 \rightarrow d = 17$$

$$a_n = 40 + (n-1)17 = 208$$

$$17n - 17 = 168$$

$$17n = 185$$

$$\boxed{n = 11}$$

$$101, \dots, 997$$

عدد سه رقمی که تقسیم بر ۷ باقی مانده ۳ دارد، ۹۹۷

$$\text{تعداد} = \frac{997 - 101}{7} + 1 = \sqrt{129}$$

$$100 \leq 7n + 3 \leq 999$$

$$97 \leq 7n \leq 996$$

$$14 \leq n \leq 142, \dots$$

$$14, \dots, 142$$

$$142 - 14 + 1 = \sqrt{129}$$

$$a_{n+1} + a_{n+2} + a_{n+3} = -4n + 12$$

$$a_{14} + a_n = ?$$

$$\rightarrow a_1 + nd + a_1 + (n+1)d + a_1 + (n+2)d = 3a_1 + (3n+3)d = -4n + 12$$

$$\Rightarrow 3d = -4 \rightarrow d = -\frac{4}{3} \quad / \quad 3a_1 + 3d = 12 \rightarrow 3a_1 + 3 \times -\frac{4}{3} = 12 \rightarrow 3a_1 = 16, \boxed{a_1 = \frac{16}{3}}$$

$$a_1 + a_{14} = \left[\frac{16}{3} + 14 \times \left(-\frac{4}{3}\right) \right] + \left[\frac{16}{3} + 14 \times \left(-\frac{4}{3}\right) \right] = -\frac{16}{3} - \frac{112}{3} = \sqrt{-48}$$

$$\begin{array}{ccc} 2^x - 1 & , & 2^{x+1} \\ \downarrow & & \downarrow \\ a_3 & & a_n \\ \downarrow & & \downarrow \\ 3 & & 1 \end{array} \quad , \quad \begin{array}{ccc} 2^{2x} - 3 & & \\ \downarrow & & \\ a_{13} & & \\ \downarrow & & \\ 13 & & \end{array}$$

$$a_3 + a_1 + a_{13} = 3 + 1 + 11 = \sqrt{15}$$

$$a_1 + a_n = a_{13} + a_3$$

$$2^{x+1} + 2^{x+1} = 2^{2x} - 3 + 2^x - 1 = 2^{2x} + 2^x - 4$$

$$2 \times 2^{x+1} = 2^{2x} + 2^x - 4$$

$$2^{x+2} - 2^{x+1} - 2^x = -4$$

$$2^x (2^2 - 2^1 - 1) = -4$$

$$2^x (4 - 2 - 1) = -4 \rightarrow \boxed{x = 2}$$

$$a_{14} - a_{10} = 5$$

$$a_{10} + a_{14} = 25$$

$$a + 11d - a - 9d = 5 \rightarrow 2d = 5 \quad d = \frac{5}{2}$$

$$a + 11d + a + 9d = 25$$

$$2a + 20d = 25$$

$$2a = 25 - 50 = -25 \quad a = -12,5$$

$$a_{21} = a + 20d = -12,5 + 20 \times \frac{5}{2} = \sqrt{37,5}$$

$$a_1 + a_2 + \dots + a_n = \frac{1}{r} (a_1 + a_2 + \dots + a_n)$$

$$\frac{a_2}{a_1} = ?$$

$$x-d-n \cdot \Delta x = \frac{1}{r} (x + r d + x + r d + \dots + x + r d)$$

$$\Delta x = \frac{1}{r} (x + r d)$$

$$1 \Delta x = \Delta x + r d$$

$$1 \Delta x = r d$$

$$x = r d$$

$$\frac{a_2}{a_1} = \frac{x-d}{x-rd} = \frac{r d - d}{r d - r d} = \frac{r d}{r d} = 1$$

$$\frac{r}{1-r} = \frac{r}{r} = 1$$

$$t_n = t_{n-1} + 1 \Delta$$

$$\frac{t_9 - t_8}{t_9} = ? \sqrt{\frac{r_0}{r_0}}$$

$$\rightarrow t_1 + (n-1)d = t_1 + (n-1)d + 1 \Delta \rightarrow r d = 1 \Delta \rightarrow d = \Delta$$

$$t_8 = t_1 + r d = t_1 + r_0$$

$$t_9 = t_1 + 9d = t_1 + r_0$$

$$t_9 = t_1 + 1 \Delta = t_1 + r_0$$

$$t_9 - t_8 = (t_9 - t_8)(t_9 + t_8) = r_0 (t_1 + r_0 + t_1 + r_0) = 2 r_0 (t_1 + r_0) = 2 r_0 \underbrace{(t_1 + r_0)}_{t_9}$$

$$r(1 - \sqrt{r}), r + \sqrt{r}$$

$$d = \frac{r + \sqrt{r} (r - 1 \sqrt{r})}{1 + 1} = \frac{\sqrt{r} + 1 \sqrt{r}}{1 + 1} = \frac{2 \sqrt{r}}{2} = \sqrt{r}$$

$$f(n-1) + r = f(n)$$

$$f(n-1) - 1 = f(n-2)$$

$$d = \frac{r-1}{f(n-2)} = \frac{r_0}{f(n-2)} = \frac{1 \Delta}{f(n-2)}$$

$$a_{n+1} = r + n d = 11 \rightarrow n \times \frac{1 \Delta}{f(n-2)} = 9 \rightarrow 1 \Delta n = 1 \Delta n - 9$$

$$f(n) = 9 \rightarrow n = 10$$

$$11 \leftarrow n$$

$$f(n-1) \leftarrow 10, f(n-2) \leftarrow 9$$

$$a_n = r a_{n-1}$$

$$a_r = r a_1$$

$$a_r = r a_r$$

$$a_1 = r a_r$$

$$a_n + a_{n+r} = a_1 + (n-1)d + a_1 + (n+r)d = 2a_1 + (2n+r)d$$

$$a_r + a_{10} = a_1 + r d + a_1 + 10d = 2a_1 + 11d$$

$$2a_1 + (2n+r)d = 2a_1 + 11d \rightarrow \underbrace{(2n+r)d}_{11d} = 11d \Rightarrow n = 10$$

$$a + r d = r (a + r d) = r a + r^2 d \rightarrow -r a - r^2 d = 0 \rightarrow a + r d = 0$$

$$a_k = a + (k-1)d = -d + (k-1)d = (k-2)d$$

$$\rightarrow d \neq 0 \rightarrow k-2 = 0 \rightarrow k = 2$$