۲۰

به نام خدا - کوثر عابدینی / A1 تکلیف نگارش ۱۲

سوال ۱

$a_1 = 40$, $a_4 = 61$ $\Rightarrow d = \frac{61-40}{4-1} = \frac{21}{3} = 7$

$a_n = 7n + 33 \rightsquigarrow a_? = 208$

$\Rightarrow 7n + 33 = 208 \Rightarrow 7n = 175 \Rightarrow n = 25$

سوال ۲

$100 \leq 7k + 3 \leq 999$

$\Rightarrow 97 \leq 7k \leq 996 \Rightarrow 13.8 \leq k \leq 142.2$

$\Rightarrow 14 \leq k \leq 142 \Rightarrow (142 - 14) + 1 = 129$

سوال ۳

$n=1 \Rightarrow n_2 + n_3 + n_4 = -6 + 12$

$\Rightarrow 3a_1 + 9d = 6 \Rightarrow a_1 + 2d = 2$ ✓

$n=2 \Rightarrow n_3 + n_4 + n_5 = -12 + 12$

$\Rightarrow 3a_1 + 9d = 0 \Rightarrow a_1 + 3d = 0$ ✓

$\begin{cases} -a_1 - 2d = -2 \\ a_1 + 3d = 0 \end{cases}$

$d = -2 \Rightarrow a_1 = 6$

$a_{17} + a_8 = (a_1 + 16d) + (a_1 + 7d) = 2a_1 + 23d$

$\Rightarrow 2 \times 6 + 23 \times (-2) = 34$

سوال ۴

$\underbrace{2^{2n} - 3}_{a_{13}}, \underbrace{2^{n+1}}_{a_8}, \underbrace{2^n - 1}_{a_3}$; $5d$, $5d$ $\Rightarrow a_8 - a_3 = a_{13} - a_8$

$2^{n+1} - 2^n + 1 = 2^{2n} - 3 - 2^{n+1}$; $2^{n+1} \Rightarrow 2 \times 2^n$

$\Rightarrow 2^n + 1 = 2^{2n} - 2 \times 2^n - 3 \Rightarrow 2^{2n} - 3 \times 2^n - 4 = 0$

$2^n = y \Rightarrow y^2 - 3y - 4 = 0 \rightsquigarrow y = \frac{3 \pm \sqrt{25}}{2}$

$y_1 = 4 \Rightarrow 2^n = 4 \Rightarrow n = 2$

$y_2 = -1$ ×

$a_3 = 2^2 - 1 = 3$ ، $a_8 = 2^{2+1} = 8$

$a_{13} = 2^{2 \times 2} - 3 = 13$

$\Rightarrow 3 + 8 + 13 = 24$

سوال ۵

$\begin{cases} a_{12} - a_{10} = 5 \\ a_{12} + a_{10} = 25 \end{cases}$; $2d = 5$

$2a_{12} = 30 \Rightarrow a_{12} = 15 \Rightarrow a_{10} = 10$

$a_{12}, a_{14}, a_{16}, a_{18}, a_{20}, a_{21}$

$15, 20, 25, 30, 35, 35 + \frac{5}{2} = 37.5$

سوال ۶

$S_5 = \frac{5}{2}(2a_1 + (5-1)d)$

$S_{10} = \frac{10}{2}(2a_1 + (10-1)d)$

$4S_5 = S_{10} \Rightarrow 4 \times [\frac{5}{2}(2a_1 + 4d)] = 5(2a_1 + 9d)$

$\Rightarrow 10(2a_1 + 4d) = 5(2a_1 + 9d)$

$\Rightarrow 2(a_1 + 4d) = 2a_1 + 9d \Rightarrow 4a_1 + 8d - 2a_1 - 9d = 2a_1 - d$

$\Rightarrow d = 2a_1$

$\frac{a_2}{a_1} = \frac{a_1 + d}{a_1} = \frac{a_1 + 2a_1}{a_1} = \frac{3a_1}{a_1} = 3$

سوال ۷

$t_n \,/\, t_n = t_{n-3} + 15$

$\rightsquigarrow n - (n-3) = 3$: از جمله t_{n-3} تا جمله t_n سه تا d فاصله داریم:

$\Rightarrow t_n = t_{n-3} + 3d \Rightarrow 3d = 15 \Rightarrow d = 5$

$t_9^2 - t_5^2 = (t_9 - t_5)(t_9 + t_5)$

$\Rightarrow (t_1 + 40 - t_1 - 20)(t_1 + 40 + t_1 + 20)$

$\Rightarrow 20 \times (2t_1 + 60) = 40t_1 + 1200$

$\Rightarrow t_7 = t_1 + (7-1)d = t_1 + 6d = t_1 + 30$; 6×5

$\frac{t_9^2 - t_5^2}{t_7} = \frac{40t_1 + 1200}{t_1 + 30} = \frac{40(t_1 + 30)}{t_1 + 30} = 40$

سوال ۸

$d = \frac{b - a}{n + 1}$

$d = \frac{2 + \sqrt{3} - 2 + 12\sqrt{3}}{13} = \frac{13\sqrt{3}}{13} = \sqrt{3}$

سوال ۹

$d = \frac{32 - 2}{4n - 3 + 1} = \frac{30}{4n - 2} = \frac{15}{2n - 1}$

واسطه n جمله n+1 ام دنباله است (a_{n+1})

$a_{n+1} = 11$

$\Rightarrow a_k = a_1 + (k-1)d \Rightarrow 11 = 2 + (n)d \Rightarrow 9 = \frac{15n}{2n-1}$

$\Rightarrow 18n - 9 = 15n \Rightarrow 3n = 9 \Rightarrow n = 3$

سوال ۱۰

$a_n + a_{n+4} = a_3 + a_{17}$

$\Rightarrow n + n + 4 = 3 + 17 \Rightarrow n = 8$

$a_n + a_{12} = a_3 + a_{17}$ ، $a_8 = 3 \times a_4$; $1 \leftrightarrow 3 \times a_{\frac{n}{2}} = a_n$

$\Rightarrow a_1 + 7d = 3a_1 + 9d \Rightarrow a_1 = -d$

$0 = a_1 + (k-1)d \Rightarrow -2d + kd = 0 \Rightarrow k = 2$

$\Rightarrow a_2 = 0$