

$$S_n = \frac{A}{r} (ra + (n-1)d)$$

$$S_0 = \frac{A}{r} (ra + rd)$$

$$S_{10} - S_0 = \frac{A}{r} (ra + 9d) - \frac{A}{r} (ra + rd)$$

$$\frac{1}{r} (10a + 45d) = \frac{A}{r} (ra + rd)$$

$$10a + 45d = 10ra + 10rd$$

$$\frac{a}{d} = \frac{r_0}{r} = r$$

$$10d = 10ra$$

$$d = ra$$

$$a_r = a_1 + d \rightarrow a_1 + ra_1 = ra_1$$

$$A (ra + 9d) - \frac{A}{r} (ra + rd) = A (ra + 9d) - A (a + rd) = 10ra + 45d - 10a - 10d = 10a + 35d$$

$$t_n = t_{n-1} + 10$$

$$t_{10} = t_1 + 10(10-1) = t_1 + 90$$

$$t_{n-1} = t_1 + (n-1)d$$

$$t_{10} = t_1 + 10(10-1) = t_1 + 90$$

$$t_{10} = t_1 + (10-1)d + 10 = t_1 + 9d + 10$$

$$t_9 = t_1 + 10(9-1) = t_1 + 80$$

$$t_1 + (10-1)d = t_1 + 9d + 10$$

$$t_9 = t_{10} = (t_9 - t_{10})(t_9 - t_{10}) =$$

$$(10-1)d = (9-1)d + 10 \rightarrow 9d = 8d + 10 \rightarrow d = 10$$

$$r_0 (r_0 + 90) = 10r_0 t_1 + 10r_0$$

$$\frac{r_0 t_1 + 10r_0}{t_1 + r_0} = \frac{r_0 (t_1 + 10)}{t_1 + r_0} = r_0$$

$$d = \frac{r + \sqrt{r} - r + 10\sqrt{r}}{1r + 1} = \frac{10\sqrt{r}}{1r + 1} = \sqrt{r}$$

$$r(1 - 10\sqrt{r}) = r - 10r\sqrt{r}$$

$$d = \frac{r\sqrt{r} - r}{r\sqrt{r} - r + 1} = \frac{r_0}{r\sqrt{r} - r + 1} \rightarrow \frac{r_0}{r(r\sqrt{r} - r + 1)} = \frac{10}{r\sqrt{r} - r + 1}$$

$$a_{n+1} = a_n + nd$$

$$11 = r + n \left(\frac{10}{r\sqrt{r} - r + 1} \right) \rightarrow a = \frac{10n}{r\sqrt{r} - r + 1}$$

$$r\sqrt{r} - r = 10n$$

$$r\sqrt{r} = 10n + r$$

$$n = r$$

$$a_n + a_{n+1} + f = a_r + a_{r+1}$$

$$a_n = a_1 + (n-1)d = 0$$

$$a_n = -d + nd - d = 0$$

$$n + n + f = r_0 \rightarrow r\sqrt{r} = 10 \rightarrow n = 1$$

$$a_n = -rd + nd$$

$$a_1 = r\sqrt{r} \rightarrow a_1 = r\sqrt{r}$$

$$-rd + nd = 0$$

$$a_1 + rd = r\sqrt{r} + 10d \rightarrow -r\sqrt{r} = 10d \rightarrow a_1 = -d$$

$$nd = rd$$

$$n = r$$

$$\begin{aligned} a_1 &= f_0 \rightarrow a + d = f_0 \quad \times \rightarrow -d = -f_0 \\ a_{10} &= 91 \rightarrow a + 9d = 91 \quad \times \rightarrow +9d = 91 \end{aligned}$$

$$t_n = f_0 + (n-1)d \quad \checkmark \quad \text{ب.}$$

$$\begin{aligned} 9d &= 91 - f_0 \\ d &= \frac{91 - f_0}{9} \end{aligned}$$

$$t_n = f_0 + (n-1)d$$

$$f_0 + (n-1)d = 91$$

$$9d = 91 + f_0 - f_0$$

$$9d = 91$$

$$d = \frac{91}{9}$$

ب. ✓

$$12 \times 9 = 108 \rightarrow 108 + 1 = 109 \rightarrow 101, 109, 117, \dots, 991$$

$$109 = \frac{991 - 101}{9} + 1 = 129 \quad \text{ب. ✓}$$

$$a_{n+1} + a_{n+2} + a_{n+3} = -2n + 12$$

$$n=10 \rightarrow a_{11} + a_{12} + a_{13} = -20 + 12 = -8$$

$$a_{11} + a_{12} + a_{13} = a_{11} + a_{12} + a_{13} + a_{14} + a_{15} + a_{16} + a_{17}$$

$$n=11 \rightarrow a_{12} + a_{13} + a_{14} = -22 + 12 = -10$$

$$2(a_{12} + a_{13}) = -10$$

$$a_{11} + a_{12} + a_{13} + a_{14} + a_{15} + a_{16} + a_{17} = -10$$

$$a_{12} + a_{13} = -5 \quad \text{ب. ✓}$$

$$\frac{r^x - 1}{r - 1} = \frac{r^{x+1} - 1}{r - 1} + \frac{r^x}{r - 1}$$

$$r^x + 1 = r^x(r + 1)$$

$$a_{10} = r^9 = 10^9$$

$$r^x r + 1 = r^x(r + 1)$$

$$a_{11} = r^{10} = 10^{10}$$

$$a_{10} + a_{11} = a_{10} + a_{11}$$

$$r^x r + 1 = r^x(r + 1)$$

$$a_{12} = r^{11} = 10^{11}$$

$$r^x - r^{x+1} = -r^{x+1} \rightarrow r^x(1 - r) = -r^{x+1} \rightarrow r^x = r^{x+1} \rightarrow r = 2$$

$$r^x = r^{x+1} \rightarrow r = 2$$

$$a_{13} = 10^{12} + 10^{11} = 1.1 \times 10^{12}$$

$$a_{12} - a_{10} = a \rightarrow 10^{11} - 10^9 = a \rightarrow a = \frac{10^{11} - 10^9}{1}$$

$$a_{12} + a_{10} = 10^{11} + 10^9$$

ب. ✓

$$10^{11} + 10^9 = 10^9(10^2 + 1) = 10^9(101) \rightarrow a_{12} = 101 \times 10^9$$

$$a_{11} = a_1 + 10d \rightarrow \frac{101 \times 10^9}{10} + 10 \times \left(\frac{101 \times 10^9}{9}\right) = \frac{101 \times 10^9}{9}$$