به نام خدا

زهرا سهرابی

تکلیف ۱۳ — ۲۰

(۱

$a_1=40 \quad a_4=61$

$a_1+3d=61 \rightarrow 40+3d=61 \rightarrow 3d=21 \rightarrow d=7$

$a_n=a_1+(n-1)7 \rightarrow 40+(n-1)7=208$

$40+7n-7=208 \rightarrow 7n=175 \rightarrow n=25$

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$14\times7=98+3=101$ اولین عدد

$7\times142=994+3=997$

$101,\ 108,\ 115,\ \dots,\ 997$ ($+7$)

$\dfrac{\text{آخری} - \text{اولی}}{7}+1$ تعداد $\rightarrow \dfrac{997-101}{7}+1=129$

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(۳

$a_{n+1}+a_{n+2}+a_{n+3}=-6n+12$

$n=10 \rightarrow a_{11}+a_{12}+a_{13}=-48$

$n=11 \rightarrow a_{12}+a_{13}+a_{14}=-54$

$a_{11}+a_{12}+a_{13}+a_{12}+a_{13}+a_{14}=-102$

$a_{11}+a_{14}=a_{12}+a_{13}=a_{13}+a_{12}=a_{17}+a_8$

$3(a_{17}+a_8)=-102$

$a_{17}+a_8=-34$

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(۴

$\underbrace{2^{2x}-3}_{a_{13}}$ و $\underbrace{2^{x+1}}_{a_8}$ ، $\underbrace{2^x-1}_{a_3}$

$a_{13}+a_3=a_8+a_8$

$2^{2n}-3+2^n-1=2^{n+1}+2^{n+1} \rightarrow 2\times2^{n+1}$

$2^{2n}+4=2^n(2^2+1)$

$2^{2n}+2^2+4=2^n(2^n+1)$

$4\times2^n+4=2^n(2^n+1)$

$4(2^n+1)=2^n(2^n+1)$

$2^n=4$

$n=2$

$a_3=2^2-1=3$

$a_8=2^{2+1}=8$

$a_{13}=2^{2(2)}-3=13$

مجموع $=3+8+13=24$

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$a_{12}-a_{10}=5 \rightarrow 2d \rightarrow d=\dfrac{5}{2}$

$a_1+11d-a_1-9d=2d$

$a_{12}+a_{10}=25$

$2a_1+20d=25 \rightarrow 2a_1+10\left(\dfrac{5}{2}\right)=2a_1+50=25 \rightarrow 2a_1=-25 \rightarrow a_1=\dfrac{-25}{2}$

$a_{21}=a_1+20d \rightarrow -\dfrac{25}{2}+20\left(\dfrac{5}{2}\right)=\dfrac{75}{2}$

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(۶

$S_n=\dfrac{n}{2}(2a+(n-1)d)$

$S_5=\dfrac{5}{2}(2a+4d)$

$S_{10}-S_5=\dfrac{10}{2}[2a+9d]-\dfrac{5}{2}[2a+4d]$

$5(2a+9d)-\dfrac{5}{2}(2a+4d)=5(2a+9d)-5(a+2d)=10a+45d-5a-10d=5a+35d$

$\dfrac{1}{3}(5a+35d)=\dfrac{5}{2}(2a+4d)$

$10a+70d=30a+60d$

$10d=20a$

$d=2a$

$a_2=a_1+d \rightarrow a_1+2a_1=3a_1$

$\dfrac{a_2}{a_1}=\dfrac{3a_1}{a_1}=3$ برابر

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$t_n=t_{n-3}+15$

$t_{n-3}=t_1+(n-1)d$

$t_n=t_1+(n-3-1)d+15$

$t_1+(n-1)d=t_1+(n-4)d+15$

$(n-1)d-(n-4)d=15 \rightarrow 3d=15 \rightarrow d=5$

$t_5=t_1+5(5-1)=t_1+20$

$t_7=t_1+5(7-1)=t_1+30$

$t_9=t_1+5(9-1)=t_1+40$

$t_9^2-t_5^2=(t_9-t_5)(t_9+t_5)$

$=(t_1+40-t_1-20)(t_1+40+t_1+20) \rightarrow 20(2t_1+60)=40t_1+1200$

$\dfrac{40t_1+1200}{t_1+30}=\dfrac{40(t_1+30)}{t_1+30}=40$ برابر

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(۸

$d=\dfrac{b-a}{n+1} \rightarrow \dfrac{2+\sqrt{3}-2+12\sqrt{3}}{13}=\dfrac{13\sqrt{3}}{13}=\sqrt{3}$

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$d=\dfrac{32-2}{4n-3+1}=\dfrac{30}{4n-2} \rightarrow \dfrac{30}{2(2n-1)}=\dfrac{15}{2n-1}$

$a_{n+1}=a_1+nd$

$11=2+n\left(\dfrac{15}{2n-1}\right) \rightarrow 9=\dfrac{15n}{2n-1} \rightarrow 18n-9=15n$

$3n=9$

$n=3$

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10)

$a_n + a_{n+4} = a_3 + a_{17}$

$n + n + 4 = 20 \rightarrow 2n = 16 \rightarrow n = 8$

$a_8 = 3a_{\frac{n}{2}} \rightarrow a_8 = 3a_4$

$a_1 + 7d = 3a_1 + 9d \rightarrow -2a_1 = 2d \rightarrow a_1 = -d$

$a_n = a_1 + (n-1)d = 0$

$a_n = -d + nd - d = 0$

$a_n = -2d + nd$

$-2d + nd = 0 \rightarrow nd = 2d$

$n = 2$

(2) ✓