پرنیان باقری / پاسخنامه تشریحی تکلیف شماره ۱۳ / کلاس دهم دختر A

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$a_1 = 40 ,\ a_4 = 61 \rightsquigarrow a_4 - a_1 = a_1 + 3d - a_1 = 3d = 61 - 40 = 21 \Rightarrow d = 7 \Rightarrow a_n = 7n + 33$

$7n + 33 = 208 \rightsquigarrow n = 25$

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$\cancel{100} \leq 7n + 3 \leq \cancel{999}$ (101, 997) $\rightsquigarrow 14 \leq n \leq 142 \Rightarrow 142 - 14 + 1 = 129$

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$a_{n+1} + a_{n+2} + a_{n+3} = -6n + 12 \rightsquigarrow 3a_{n+2} = -6n + 12 \rightsquigarrow a_{n+2} = -2n + 4 \rightsquigarrow a_n = -2(n-2) + 4 = -2n + 8$

$a_8 = -2(8) + 8 = (-8)$

$a_{17} = -2(17) + 8 = (-26)$

$\Rightarrow a_8 + a_{17} = (-8) + (-26) = (-34)$

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$2^x - 1 \ (3),\ 2^{x+1} \ (8),\ 2^{3x} - 3 \ (13) \rightsquigarrow 2^{x+1} - 2^x + 1 = 2^{3x} - 3 - 2^{x+1} \rightsquigarrow 2^x(2 - 1 + 2^{x-2}) = 2^x(2^x - 2)$

$1 + 2^{x-2} = 2^x - 2 \rightsquigarrow 2^{x-2}(-1 + 2^2) = 3 \rightsquigarrow 2^{x-2} = 1 \rightsquigarrow x - 2 = 0 \Rightarrow x = 2$

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$\begin{cases} a_{12} - a_{10} = 5 \\ a_{12} + a_{10} = 25 \end{cases} \rightsquigarrow a_{10} + 2d - a_{10} = 2d = 5 \rightsquigarrow d = \frac{5}{2}$

$a_{12} = 15 \rightsquigarrow a_{21} = a_{12} + 9d = 15 + \frac{45}{2} = \frac{75}{2} = 37.5$

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$3 \times \frac{5}{2}(a_1 + a_5) = \frac{5}{2}(a_4 + a_{10}) \rightsquigarrow 3a_1 + 3a_5 = a_4 + a_{10} \rightsquigarrow 3a_1 + 3a_1 + 12d = a_1 + 5d + a_1 + 9d$

$6a_1 + 12d = 2a_1 + 14d$

$(2a_1 = d)$

$\frac{a_2}{a_1} = \frac{a_1 + d}{a_1} = \frac{3a_1}{a_1} = 3$

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$\frac{t_9^2 - t_5^2}{t_7} = \frac{(t_9 + t_5)(t_9 - t_5)}{\frac{t_5 + t_9}{2}} = 2\underbrace{(t_9 - t_5)}_{4d} = 8d = 8(5) = 40$

$\left[t_n = t_{n-3} + 15 \rightsquigarrow t_4 = t_1 + 15 \rightsquigarrow t_4 - t_1 = 3d = 15 \rightsquigarrow d = 5 \right]$

$(t_4 = t_1 + 3d)$

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$d = \frac{t_m - t_1}{n - 1} \rightsquigarrow \frac{2 + \sqrt{3} - 2 + 12\sqrt{3}}{12 + 1} = \frac{13\sqrt{3}}{13} = \sqrt{3}$

+1 تعداد واسطه حسابی

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$a_1 = 2 ,\ a_{n+1} = 11 ,\ a_{4n-1} = 32$

$d = \frac{a_{n+1} - a_1}{n + 1 - 1} = \frac{11 - 2}{n} = \frac{9}{n} ,\ d = \frac{a_{4n-1} - a_{n+1}}{4n - 1 - n - 1} = \frac{32 - 11}{3n - 2} = \frac{21}{3n - 2} \Rightarrow \frac{9}{n} = \frac{21}{3n - 2}$

$27n - 18 = 21n \rightsquigarrow n = 3$

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$a_n + a_{n+4} = a_3 + a_{17} \rightsquigarrow n + n + 4 = 13 + 7 \rightsquigarrow 2n + 4 = 20 \rightsquigarrow n = 8$

$a_n = 3a_{\frac{n}{2}} \rightsquigarrow a_8 = 3a_4 \rightsquigarrow a_8 - a_4 = a_4 + 4d - a_4 = 4d = 2a_4 \rightsquigarrow 2d = a_4 \rightsquigarrow \boxed{a_2 = 0}$