عارف سعادت نیا

۱) الف) $t_{10} = t_1 \times q^9 = \frac{1}{2} \times 2^9 = 2^8$ ۲۵۶

$a_4 = \frac{1}{2} \times 2^3 = 4$ $a_{10} = \frac{1}{2} \times 2^9 = 256$

$\sqrt{4 \times 256} = \sqrt{1024} = 32$ واسطه هندسی

ب) $\frac{t_1 q_1^{16}}{t_1 q^{12}} = \frac{\frac{1}{2} \times 2^{19}}{\frac{1}{2} \times 2^{12}} = 2^6 = 64$

ج) $\frac{1}{2} \times q^{n-1} = 128$ $q^{n-1} = 128 \times 2 = 256$

$2^8 = 256$ $n-1 = 8$ $n = 9$

د) $\frac{t_8}{t_5} = q^3 = 2^3 = 8$ $q = 2$

$t_{10} = t_1 \times 2^9 = \frac{1}{2} \times 2^9 = 256$

$t_{10} = t_1 \times q^9$

$64 \times 4 = 256$

۲) الف) $a_1 \times a_2 \times a_3 \times a_4 \times a_5 = (a_3)^5 = 243$

$a_3 = 3$

$\frac{a_2}{q^2} \times \frac{a_3}{q} \times a_2 \times a_4 q \times a_3 q^2 = 243$

$5 a_3 = 243$

$a_3 = 48.6$

$a_1 \times a_5 + (a_3)^2 = 9$

$a_1 \times a_5 = \frac{9}{2} \times a_3 q^2 = 2 a_3 = 2 \times 48.6 = 97.2$

ب) $\frac{a_7}{a_2} = \frac{a_{11}}{a_7}$ $\frac{x}{1-x} = \frac{x+1}{x}$ $x^2 = (1-x)(x+1)$

$x^2 = -x^2 + 1$ $2x^2 = 1$ $x^2 = \frac{1}{2}$ $x = \pm \frac{1}{\sqrt{2}}$

فقط یک دنباله!

۳) $a_1 + a_2 + a_3 = 13$

$\frac{a_2}{2} + a_1 + a_2 2 = 13$

$\frac{3}{2} + 3 + 3q = 13$

$q = 2$

۴) $\sqrt[5]{2^2} = 2^a \times 2^b$ $a + b = 5$

$2^{\frac{2}{5}} = 2^a \times 2^b$ $\frac{a+b}{2} = \frac{5}{2} = 2.5$

۵) $a_1 + a_1 q^3 = 28$

$a_1 q + a_1 q^2 = 12$

$3 + 3q^3 = 7q + 7q^2$

$q = 3$

۶) $\frac{a_1(1+q^3)}{a_1(q+q^2)} = \frac{28}{12}$

۷) $a_1 \times a_2 \times a_3$

$\frac{a_2}{2} \times a_2 \times a_2 2 = 27$

$a_2^3 = 27$

$a_2 = 3$

۸) $1, 3, 9, \ldots$

$9, 3, 1, \ldots$

۹) $a(q-1)[1, q, q^2, \ldots]$ (دنباله هندسی)

قدرنسبت جمله $= q$

۱۰) الف) $S_{10} = 2 \times \left[\frac{3^{10} - 1}{\frac{3-1}{2}}\right] = 59048$

ب) $2^{10} \times 2^{\frac{n(n-1)}{2}} = 2^{10} \times 3^{45}$

ج) $S_n = a_1 + a_2 + a_3 + \cdots + a_n$

$S_n = a_n + a_{n-1} + a_{n-2} + \cdots + a_1$

$2S_n = (a_1 + a_n) + (a_2 + a_{n-1}) + (a_3 + a_{n-2}) + \cdots$ (n ترم)

$S_n = \frac{n}{2}(a_1 + a_n)$ $\rightarrow a_n = a_1 + (n-1)d$

$S_n = \frac{n}{2}(2a_1 + (n-1)d)$

$S_n = a + aq + aq^2 + \cdots + aq^{n-1}$ $\times q$

$qS_n = aq + aq^2 + aq^3 + \cdots + aq^n$

$qS_n - S_n = aq^n - a$

$S_n(q-1) = a(q^n - 1)$

$S_n = a\left(\frac{q^n - 1}{q-1}\right)$

$a_1 + a_2 + a_3 + \cdots + a_n = a_1 + a_1 q + a_1 q^2 + \cdots + a_1 q^{n-1}$

$= a_1 \left(\frac{q^n - 1}{q-1}\right)$

$a_1 + a_2 + a_3 + \cdots + a_n = a_1 + a_1 q + a_1 q^2 + \cdots + a_1 q^{n-1}$

$= a_1 \left(\frac{1 - q^n}{1-q}\right)$

$a q - a = a(q-1)$

$aq^2 - aq = aq(q-1)$

$aq^3 - aq^2 = aq^2(q-1)$