۱

$a_1+a_2+a_3=21$ و $a_1\times a_2\times a_3=64 \rightsquigarrow \frac{a_2}{q}\times a_2\times a_2q = a_2^3 = 64 \Rightarrow a_2=4$

$a_1\times a_3 = 16 \rightsquigarrow a_1^2q^2=16$

$a_1+a_3=17 \rightsquigarrow a_1+a_1q^2=17 \xrightarrow{a_1q^2=\frac{16}{a_1}} a_1+\frac{16}{a_1}=17 \rightsquigarrow a_1^2-17a_1+16=0 \rightsquigarrow a_1=1$ یا 16

$a_1=1,\ a_2=4 \rightsquigarrow q=4$ غ ق ق $(q\notin W)$

$a_1=16,\ a_2=4 \rightsquigarrow \boxed{q=\frac{1}{4}\ ✓}$

۲

$x^2+4,\ 2x,\ x^2-2 \rightsquigarrow 4x^2=(x^2+4)(x^2-2)=x^4+2x^2-8 \rightsquigarrow x^4-2x^2-8=0 \rightsquigarrow (x^2-4)\underbrace{(x^2+2)}_{\text{همواره مثبت}}=0$

$x^2-4=0 \rightsquigarrow x=\pm 2$

$x=2 \rightsquigarrow 8,4,2 \rightsquigarrow q=\frac{1}{2}$

$x=(-2) \rightsquigarrow 8,-4,2 \rightsquigarrow q=\frac{-1}{2}$ غ ق ق $(q>0)$

$S_7=8\left(\frac{1-(\frac{1}{2})^7}{1-\frac{1}{2}}\right)=16-\frac{1}{8}=\frac{127}{8}$

۳

$1+q+q^2+q^3+q^4=\frac{121}{81}\ ,\ a_1=243$

$S_5=a_1+a_1q+a_1q^2+a_1q^3+a_1q^4=a_1(1+q+q^2+q^3+q^4)=243\times\frac{121}{81}=363$

۴

$a_1=1,\ a_7=64 \rightsquigarrow a_7=a_1+6d=64 \rightsquigarrow 1+6d=64 \rightsquigarrow d=\frac{63}{6}=\frac{21}{2}=10.5 \rightsquigarrow a_4=1+3(10.5)=32.5$

$t_1=1,\ t_7=64 \rightsquigarrow t_7=t_1q^6=64 \rightsquigarrow 1\times q^6=64 \rightsquigarrow q=\pm 2 \rightsquigarrow t_4=\pm 8$

$\begin{cases}A=32.5\\ B=\pm 8\end{cases} \Rightarrow \begin{cases}A+B=32.5+8=40.5\\ A+B=32.5-8=24.5\end{cases}$

۵

$-24,\ \frac{-95}{4},\ \dots \rightsquigarrow d=\frac{-95}{4}-(-24)=\frac{1}{4} \rightsquigarrow t_{101}=t_1+100d=-24+\frac{100}{4}=1$

$a_8=t_{101}$

$128,\ a_2,\ \dots \rightsquigarrow a_8=a_1q^7=128q^7=1 \rightsquigarrow \boxed{q=\frac{1}{2}}$

۶

$a_3,\ a_7,\ a_9 \rightsquigarrow a_1+2d,\ a_1+6d,\ a_1+8d$

$(a_1+6d)^2=(a_1+2d)(a_1+8d) \Rightarrow a_1^2+36d^2+12a_1d=a_1^2+16d^2+10a_1d$

$20d^2+2a_1d=0$

$10d^2+a_1d=0 \rightsquigarrow d(10d+a_1)=0 \rightarrow \boxed{d=0}$

$10d+a_1=0 \rightarrow \boxed{d=\frac{-a_1}{10}}$

۷

$a_2,\ a_4,\ a_8 \rightsquigarrow \underbrace{a_1+d}_{2a_1},\ \underbrace{a_1+3d}_{4a_1},\ \underbrace{a_1+7d}_{8a_1} \rightsquigarrow q=2 \rightsquigarrow a_{10}=a_1q^9=\frac{1}{4}\times 2^9=\boxed{128}$

$(a_1+3d)^2=(a_1+d)(a_1+7d) \rightsquigarrow a_1^2+9d^2+6a_1d=a_1^2+7d^2+8a_1d \rightsquigarrow 2d^2-2da_1=0$

$d=a_1$

$d=0$ غ ق ق

$$۳a_۲,\ ۲a_۳,\ a_۴ \rightsquigarrow ۳a_۱q,\ ۲a_۱q^۲,\ a_۱q^۳$$

$$۲a_۱q^۲-۳a_۱q=a_۱q^۳-۲a_۱q^۲ \rightsquigarrow a_۱q(۲q-۳)=a_۱q(q^۲-۲q) \rightsquigarrow ۲q-۳=q^۲-۲q \rightsquigarrow q^۲-۴q+۳=۰$$ ۸

$$(q-۱)(q-۳)=۰ \Rightarrow q=۱ \text{ یا } ۳$$

غ ق ق $\boxed{q=۳}$

$$۲,\ \frac{۷}{۴},\ \dots \rightsquigarrow d=\frac{-۱}{۴} \qquad a_۴=۲+\frac{۳(-۱)}{۴}=\frac{۵}{۴},\ a_۸=۲+\frac{۷(-۱)}{۴}=\frac{۱}{۴},\ a_{۱۳}=۲+\frac{۱۲(-۱)}{۴}=(-۱)$$

$$a_۴+k,\ a_۸+k,\ a_{۱۳}+k \rightsquigarrow \frac{۵}{۴}+k,\ \frac{۱}{۴}+k,\ -۱+k \rightsquigarrow \left(\frac{۱}{۴}+k\right)^۲=\left(\frac{۵}{۴}+k\right)(k-۱)$$

$$\frac{۱}{۱۶}+k^۲+\frac{k}{۲}=k^۲-\frac{۵}{۴}+\frac{k}{۴} \rightsquigarrow \frac{k}{۴}=\frac{-۲۱}{۱۶} \Rightarrow k=\frac{-۲۱}{۴}$$ ۹

$$-۴,\ -۵,\ \frac{-۲۵}{۴} \Rightarrow \boxed{q=\frac{۵}{۴}}$$

$$a_۱,\ a_۴,\ a_۷ \quad ,\ a_۱+a_۴+a_۷=۷۳$$

$$a_۱=t_۱,\ a_۴=t_۲,\ a_۷=t_{۱۰} \qquad a_۴-a_۱=\frac{a_۷-a_۴}{۸} \rightsquigarrow a_۱(q^۳-۱)=\frac{a_۱q^۳(q^۳-۱)}{۸} \rightsquigarrow ۱=\frac{q^۳}{۸} \rightsquigarrow q=۲$$ ۱۰

$$a_۱+a_۱q^۳+a_۱q^۶=a_۱(۱+q^۳+q^۶)=a_۱(۱+۸+۶۴)=۷۳a_۱=۷۳ \Rightarrow a_۱=۱$$

$$a_۱=t_۱=۱ \quad a_۴=t_۲=۱\times۲^۳=۸ \rightsquigarrow t_۲-t_۱=۸-۱=\boxed{۷}$$