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الف) $(\sin 135^\circ + \sin 300^\circ)(\cos 210^\circ - \cos 120^\circ) = \left(+\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) = \frac{3}{4} - \frac{3}{4} = \frac{-1}{4}$

ب) $\frac{1+3\tan^2 210}{4\cos^2 330 + 2\cos^2 225} = \frac{1+3\times\frac{1}{3}}{4\times\frac{3}{4}+2\times\frac{2}{4}} = \frac{1+1}{3+1} = \frac{2}{4} = \frac{1}{2}$

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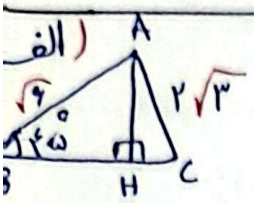
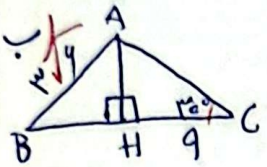
الف)



$AH = \frac{\sqrt{2}}{2} AB = \frac{\sqrt{2}}{2}\times\sqrt{6} = \sqrt{3}$ $HC^2 = AC^2 - AH^2$

$\Rightarrow HC^2 = 12 - 3 \Rightarrow HC = 3$

ب)



$\frac{AH}{HC} = \frac{1}{\sqrt{3}} = \frac{AH}{9} \Rightarrow AH = 3\sqrt{3}$ / $3\sqrt{6}\times\sin B = 2\sqrt{3}$

$\Rightarrow \sin B = \frac{\sqrt{2}}{2} \Rightarrow B = 45^\circ$ ...

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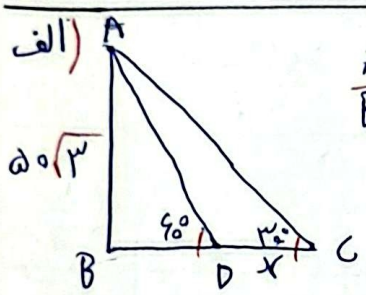
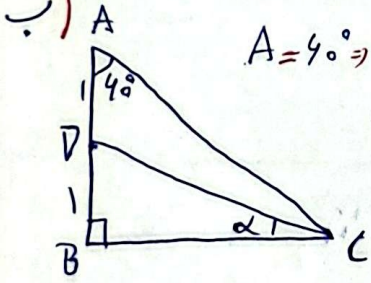
الف)



$\frac{AB}{BD} = \tan 60^\circ = \sqrt{3} \Rightarrow BD = 50$ $x = BC - BD = 150 - 50 = 100$

$\frac{AB}{BC} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow BC = 150$

ب)



$A = 60^\circ \Rightarrow C = 30^\circ \rightarrow \frac{AB}{AC} = \frac{1}{2} = \frac{2}{AC} \Rightarrow AC = 4$

$BC^2 = AC^2 - AB^2 \Rightarrow BC = 2\sqrt{3} = \sqrt{12-4}$

$CD^2 = BD^2 + BC^2 = 1 + 12 \Rightarrow \sqrt{13} = CD$

$\sin\alpha = \frac{BD}{CD} = \frac{1}{\sqrt{13}} = \frac{\sqrt{13}}{13}$

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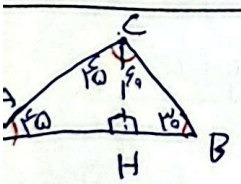
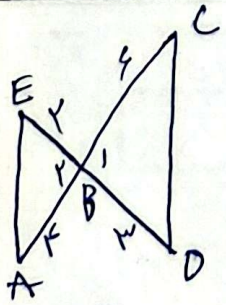


$\frac{CH}{AC} = \sin 45^\circ = \frac{\sqrt{2}}{2} \Rightarrow AC\sqrt{2} = 2CH \Rightarrow AC = CH\frac{2}{\sqrt{2}} = CH\sqrt{2}$

$\frac{CH}{AH} = \tan 45^\circ = 1 \Rightarrow CH = AH$ $\frac{CH}{BH} = \tan 30^\circ \frac{1}{\sqrt{3}} \Rightarrow BH = CH\sqrt{3}$

$\frac{AB}{AC} = \frac{AH+HB}{AC} = \frac{CH+CH\sqrt{3}}{CH\sqrt{2}} = \frac{1+\sqrt{3}}{\sqrt{2}}$

۵



$B_1 = B_2$

$S_{ABE} = \frac{1}{2}\sin B_2 \times 2 \times 4 = 4\sin B_2$

$S_{BCD} = \frac{1}{2}\times\sin B_1 \times 3 \times 6 = 9\sin B_1$

$\frac{S_{ABE}}{S_{BCD}} = \frac{4\sin B_2}{9\sin B_1} = \frac{4}{9}$

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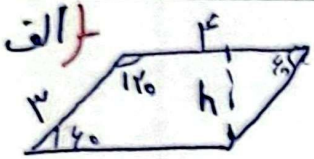
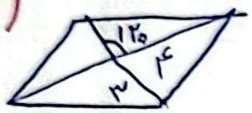
الف)



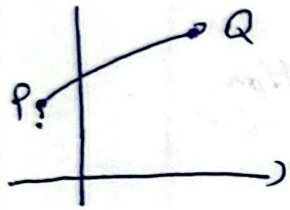
$h = \frac{\sqrt{۳}}{۲} \times ۳ \Rightarrow S = \frac{۱}{۲} \times ۴ \times \frac{۳\sqrt{۳}}{۲} = ۶\sqrt{۳}$

ب)



$S = \sin ۱۲۰ \times ۴ \times \frac{۱}{۲} = ۳\sqrt{۳}$

۷

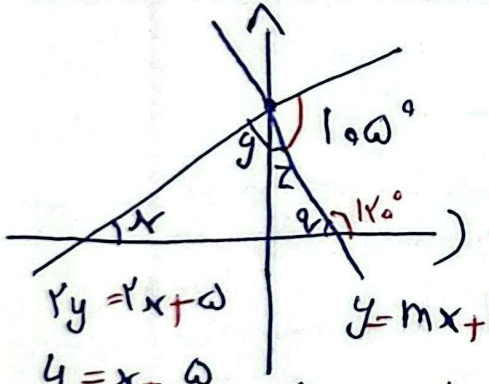


$P \to ۲ = -a + b$

$Q \to ۴ = ۳a + b \to ۴a = ۲ \Rightarrow a = \frac{۱}{۲} = \tan\theta$

$۱ + \tan^۲\theta = \frac{۱}{\cos^۲\theta} \Rightarrow \cos\theta = \frac{۲}{\sqrt{۵}}$

۸



$۲y = ۲x + ۵ \Rightarrow ۲y = ۲ \times ۰ + ۵ \Rightarrow y = ۲/۵$

$۲/۵ = m \times ۰ + b \Rightarrow b = ۲/۵$

$m = \tan ۱۲۰^\circ = \sqrt{۳} \to mb = \frac{۵\sqrt{۳}}{۲}$

$y = x + \frac{۵}{۲} \Rightarrow \tan x = ۱ \Rightarrow x = ۴۵^\circ = y$

$y + z = ۱۸۰^\circ \Rightarrow z = ۳۰^\circ \to q = ۶۰^\circ$

۹

$\cos\left(\pi + \frac{۵\pi}{۶}\right) + \sin\left(\frac{\pi}{۲} - \frac{۵\pi}{۶}\right) - \tan\left(\frac{۳\pi}{۲} + \frac{۵\pi}{۶}\right) = \cos\frac{۵\pi}{۶} + \cos\frac{۵\pi}{۶} + \cot\frac{۵\pi}{۶}$

$= \cot\frac{۵\pi}{۶} = \sqrt{۳}$

$$\frac{\sin\left(\frac{17\pi}{2}+x\right)+\cos\left(\frac{13\pi}{2}+x\right)}{\cos(3\pi+x)-\sin(4\pi-x)}$$

10

$$= \frac{\sin\left(\frac{\pi}{2}+x\right)+\cos\left(\frac{3\pi}{2}+x\right)}{\cos(\pi+x)-\sin(2\pi-x)} = \frac{\cos x-\sin x}{-\cos x+\sin x} = -1$$