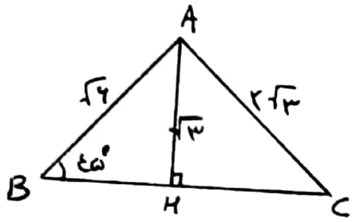
۱) الف) $(\sin 135^\circ + \sin 300^\circ)(\cos 315^\circ - \cos 150^\circ) = (\frac{\sqrt2}{2} - \frac{\sqrt3}{2})(\frac{\sqrt2}{2} + \frac{\sqrt3}{2}) = \frac{2}{4} - \frac{3}{4} = -\frac{1}{4}$

ب) $\frac{1 + 3\tan^2 210^\circ}{4\cos^2 330^\circ + 2\cos^2 225^\circ} = \frac{1 + 3 \times (\frac{1}{\sqrt3})^2}{4 \times (\frac{\sqrt3}{2})^2 + 2(-\frac{\sqrt2}{2})^2} = \frac{1+1}{3+1} = \frac{2}{4} = \frac{1}{2}$

۲) الف) طول $HC = ?$



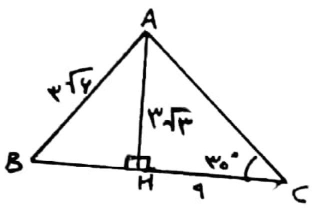
$\sin 45^\circ = \frac{\sqrt2}{2} = \frac{AH}{\sqrt6} \Rightarrow AH = \frac{\sqrt2 \times \sqrt6}{2} = \frac{\sqrt{12}}{2} = \frac{2\sqrt3}{2} = \sqrt3$

$\boxed{AH = \sqrt3}$

$HC^2 = (2\sqrt3)^2 - (\sqrt3)^2 \Rightarrow HC^2 = 12 - 3 = HC^2 = 9$

$HC = \pm\sqrt9$ طول ضلع نمی‌تواند منفی باشد $\Rightarrow \boxed{HC = 3}$

ب) زاویه $B = ?$



$\tan 30^\circ = \frac{AH}{9} = \frac{\sqrt3}{3} \Rightarrow AH = \frac{9\sqrt3}{3} = 3\sqrt3$

$\sin \hat B = \frac{3\sqrt3}{3\sqrt6} = \sqrt{\frac{3}{6}} = \frac{\sqrt2}{2} \Rightarrow \boxed{\hat B = 45^\circ}$

۳) الف) $x = ?$

$\tan 60^\circ = \sqrt3 = \frac{50\sqrt3}{BD} \Rightarrow BD = \frac{50\sqrt3}{\sqrt3} \Rightarrow \boxed{BD = 50}$

$\tan 30^\circ = \frac{\sqrt3}{3} = \frac{50\sqrt3}{BC} \Rightarrow BC = \frac{150\sqrt3}{\sqrt3} = BC = 150 \Rightarrow BC = x + 50$

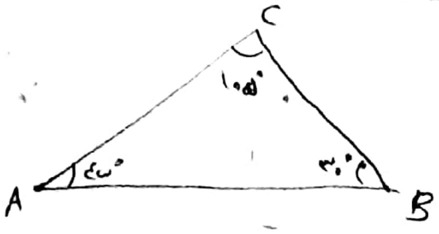
$150 = x + 50 \Rightarrow \boxed{x = 100}$

ب) $\sin\alpha = ?$

$\tan 60^\circ = \frac{BC}{2} = \sqrt3 \Rightarrow \boxed{BC = 2\sqrt3}$

$DC^2 = 1^2 + (2\sqrt3)^2 \Rightarrow DC^2 = 1 + 12 \Rightarrow \boxed{DC = \sqrt{13}} \Rightarrow \boxed{\sin\alpha} = \frac{BD}{DC} = \frac{1}{\sqrt{13}} = \boxed{\frac{\sqrt{13}}{13}}$

۴) $\frac{AB}{AC} = ?$



$\frac{BC}{\frac{\sqrt2}{2}} = \frac{AC}{\frac{1}{2}} = \frac{AB}{\frac{\sqrt6+\sqrt2}{4}} \to \frac{AB}{AC} = \frac{\frac{\sqrt6+\sqrt2}{4}}{\frac{1}{2}} = \frac{\sqrt6+\sqrt2}{2}$

$\sin 105 = \sin(60+45) = \sin 60 \cos 45 + \cos 60 \sin 45 = \frac{\sqrt3}{2} \times \frac{\sqrt2}{2} + \frac{1}{2} \times \frac{\sqrt2}{2} = \frac{\sqrt6}{4} + \frac{\sqrt2}{4} = \frac{\sqrt6+\sqrt2}{4}$

۵)

$$\frac{S_{A\hat{B}E}}{S_{B\hat{C}D}} = ? \quad \frac{۲\times۴\times\frac{۱}{۲}\times\sin\hat{B}}{۶\times۳\times\frac{۱}{۲}\times\sin\hat{B}} = \boxed{\frac{۴}{۹}}$$

۶)

الف) $۴\times۳\times\sin۱۲۰^\circ\times\frac{۱}{۲}\times۲ = ۴\times۳\times\frac{\sqrt{۳}}{۲} = \boxed{۶\sqrt{۳}}$

ب) $\frac{۱}{۲}\times۳\times۴^۲\times\sin۱۲۰^\circ = ۶\times\frac{\sqrt{۳}}{۲} = ۳\sqrt{۳}$

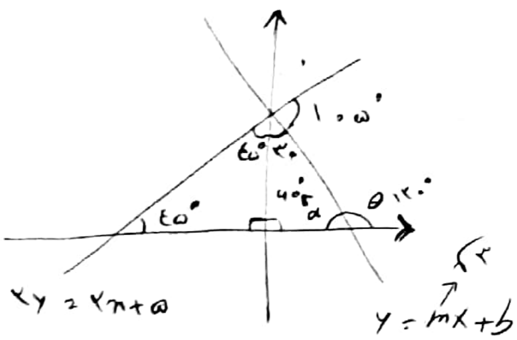
۷)

$y = ax+b$ $\quad \boxed{\text{شیب}} = \frac{۴-۲}{۳-(-۱)} = \frac{۲}{۴} = \boxed{\frac{۱}{۲}} \Rightarrow \text{شیب} = \tan\theta$

$1+\tan^۲\theta = \frac{1}{\cos^۲\theta} \Rightarrow 1+(\frac{1}{۲})^۲ = \frac{1}{\cos^۲\theta} \Rightarrow 1+\frac{1}{۴} = \frac{۵}{۴} = \frac{1}{\cos^۲\theta} \Rightarrow \cos^۲\theta = \frac{۴}{۵}$

$\boxed{\cos\theta} = \frac{۲}{\sqrt{۵}} = \boxed{\frac{۲\sqrt{۵}}{۵}}$

۸) $mb = ?$



$m\times b = \frac{۵}{۲}\times\sqrt{۳} = \frac{۵\sqrt{۳}}{۲}$

$m = \tan\beta = \tan۶۰^\circ = \sqrt{۳}$

$\beta = ۱۸۰ - (۴۵+۷۵) = ۶۰$

$y = x+\frac{۵}{۲}$

$m = 1$ شیب $\rightarrow \alpha = ۴۵^\circ$ $\quad x=0 \longrightarrow y = \frac{۵}{۲} \longrightarrow y = mx+b \rightarrow \frac{۵}{۲} = \sqrt{۳}\cdot 0 + b \rightarrow b = \frac{۵}{۲}$

۹)

$f(x) = \cos(\pi+x) + \sin(\frac{\pi}{۲}-x) - \tan(\frac{۳\pi}{۲}+x) \Rightarrow -\cos x + \cos x + \cot x$

$f(x) = \cot x \rightarrow f(\frac{۵\pi}{۶}) = \cot(\frac{۵\pi}{۶}) \Rightarrow -\sqrt{۳}$

۱۰)

$$\frac{\sin(\frac{۱۷\pi}{۲}+x)+\cos(\frac{۱۵\pi}{۲}-x)}{\cos(۳\pi+x)-\sin(۶\pi-x)} = \frac{-\cos x+\sin x}{\cos x-\sin x} = \frac{-(\cos x-\sin x)}{\cos x-\sin x} = -۱$$