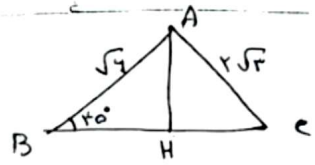
۱۹۱۷۵

۱) الف) $\left(\frac{\sqrt{2}}{2}+\frac{-\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}-\left(-\frac{\sqrt{3}}{2}\right)\right)=\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\right)=\frac{2}{4}-\frac{3}{4}=\frac{-1}{4}$

ب) $\frac{1+2\left(\frac{1}{\sqrt{2}}\right)^2}{4\times\left(\frac{\sqrt{2}}{2}\right)^2+2\left(-\frac{\sqrt{2}}{2}\right)^2}=\frac{1+1}{3+1}=\frac{2}{4}=\frac{1}{2}$

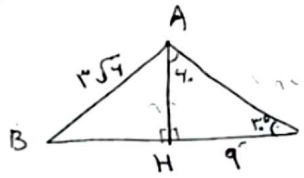
۲) الف)



$\sin 45^\circ \to \frac{\sqrt{2}}{2}=\frac{AH}{\sqrt{6}} \to AH=\frac{\sqrt{12}}{2} \to AH^2+CH^2=AC^2 \to \left(\frac{\sqrt{12}}{2}\right)^2+CH^2=(2\sqrt{3})^2$

$3+CH^2=12 \to CH^2=9 \to CH=3$

ب)



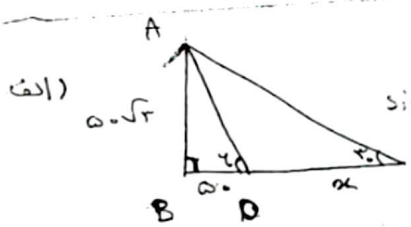
$\to \cos 30^\circ=\frac{\sqrt{3}}{2}=\frac{9}{AC} \to \frac{18}{\sqrt{3}}=AC \to 9^2+AH^2=108$

$AH^2=27 \to AH=\sqrt{27}$

$(\sqrt{27})^2+BH^2=(3\sqrt{6})^2 \to 27+BH^2=54 \to BH^2=27 \to BH=\sqrt{27}$

$\sin B=\frac{\sqrt{27}}{3\sqrt{6}} \to \frac{3\sqrt{3}}{3\sqrt{6}}=\sqrt{\frac{3}{6}}=\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2} \to$ پس $B=45^\circ$

۳) الف)



$\sin 60^\circ=\frac{\sqrt{3}}{2}=\frac{50\sqrt{3}}{AB} \to AD=100 \to (50\sqrt{3})^2+x^2=(100)^2$

$\sin 30^\circ=\frac{1}{2}=\frac{50\sqrt{3}}{AC} \to AC=100\sqrt{3}$

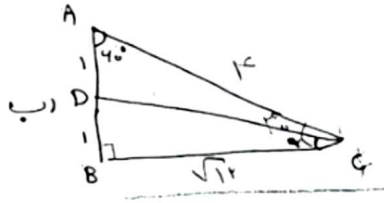
$7500+x^2=10000 \to x^2=2500$

$BD=50$

$(50\sqrt{3})^2+x^2=(100\sqrt{3})^2 \to 7500+x^2=30000 \to x=150=BC$

$CD=BC-BD=150-50=100$

ب)



$\sin 30^\circ=\frac{1}{2}=\frac{2}{AC} \to AC=4$

$\sin\alpha=\frac{1}{\sqrt{12}}$

$2^2+BC^2=16 \to 16-4=12 \to BC=\sqrt{12}$

$1+12=13 \to \sqrt{12}=CD$

۴) $\frac{AB}{AC}=?$

$45+105=150 \to 180-150=30$

$\frac{AC}{\sin B}=\frac{CB}{\sin A}=\frac{AB}{\sin C} \to \frac{AC}{\frac{1}{2}}=\frac{BC}{\frac{\sqrt{2}}{2}}=\frac{AB}{\sin 105} \Rightarrow \frac{AB}{AC}=\frac{\sin 105}{\sin 30}=\frac{\sin 60\cos 45+\sin 45\cos 60}{\frac{1}{2}}$

$\frac{\sqrt{6}}{4}$ $\quad$ $\frac{\sqrt{2}}{4}$

$\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}=\frac{\sqrt{6}+\sqrt{2}}{2}$

④ $\frac{S_{ABE}}{S_{BCD}} = \frac{\frac{1}{2}\times\sin\alpha\times 2\times 4}{\frac{1}{2}\times\sin\alpha\times 3\times 4} = \frac{8}{12} = \frac{2}{3}$

⑥ الف) $3\times 4\times\sin 120^\circ = 12\times\frac{\sqrt{3}}{2} = 6\sqrt{3}$

ب) $\frac{1}{2}\times 3\times 4\times\sin 120^\circ = \frac{1}{2}\times 12\times\frac{\sqrt{3}}{2} = 3\sqrt{3}$

$360 - 240 = 120 \div 2 = 60$

⑦ 

$4^2 + 2^2 = 16 + 4 = 20$ $\quad \cos\theta = \frac{4}{\sqrt{20}}$

⑧ $2y = 2x + 10 \Rightarrow m = 1$ $\quad \tan\theta = 1$ $\quad \theta = 45$

$y = mx + b \rightarrow m = \tan 120 = -\sqrt{3}$

$\frac{10}{2} = -\sqrt{3}(0) + b \rightarrow b = \frac{10}{2}$

$mb = \frac{5}{2}(-\sqrt{3})$

⑤ $mx + b = 0$, $2x + 5 = 0$ $\Rightarrow$ $mx + b = 2x + 5$, $m = 2, b = 5$ $\quad mb = 10$

⑨ $f(x) = -\cos\pi + \cos\pi - \cot\pi = \frac{-\cos\pi}{\sin\pi}$

$f(\frac{5\pi}{4}) = -\cot\frac{5\pi}{4} = -(-\sqrt{1}) = \sqrt{1} = -\sqrt{3}$

$\frac{5\pi}{4} \rightarrow 180^\circ$

$\frac{\sin(\frac{\pi}{2}+x) + \cos(\frac{\pi}{2}-x)}{\cos(2\pi+x) - \sin(4\pi-x)} = \frac{\cos\pi + \sin\pi}{\cos\pi - (-\sin\pi)} = \frac{-\cos\pi + \sin\pi}{\sin\pi - \cos\pi}$

⑪ 1

$\sin(\frac{11\pi}{2} + x) = \cos x$

$\cos(\frac{15\pi}{2} - x) = -\sin x$

$\cos(3\pi + x) = -\cos x$

$\sin(4\pi - x) = -\sin x$