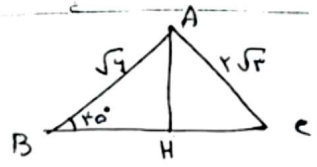
① الف) $\left(\frac{\sqrt{2}}{2}+\frac{-\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}-\left(-\frac{\sqrt{3}}{2}\right)\right)=\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\right)=\frac{2}{4}-\frac{3}{4}=\frac{-1}{4}$

ب) $\frac{1+2\left(\frac{1}{\sqrt{2}}\right)^2}{4\times\left(\frac{\sqrt{3}}{2}\right)^2+2\left(-\frac{\sqrt{2}}{2}\right)^2}=\frac{1+1}{3+1}=\frac{2}{4}=\frac{1}{2}$

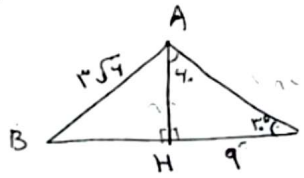
④ الف)



$\sin 45^\circ \to \frac{\sqrt{2}}{2}=\frac{AH}{\sqrt{6}} \to AH=\frac{\sqrt{12}}{2} \to AH^2+CH^2=AC^2 \to \left(\frac{\sqrt{12}}{2}\right)^2+CH^2=(2\sqrt{3})^2$

$3+CH^2=12 \to CH^2=9 \to CH=3$

ب)



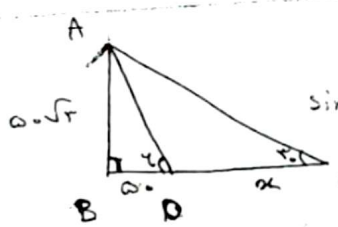
$\to \cos 30^\circ=\frac{\sqrt{3}}{2}=\frac{9}{AC} \to \frac{18}{\sqrt{3}}=AC \to 9^2+AH^2=108$

$AH^2=27 \to AH=\sqrt{27}$

$(\sqrt{27})^2+BH^2=(3\sqrt{6})^2 \to 27+BH^2=54 \to BH^2=27 \to BH=\sqrt{27}$

$\sin B=\frac{\sqrt{27}}{3\sqrt{6}} \to \frac{3\sqrt{3}}{3\sqrt{6}}=\sqrt{\frac{3}{6}}=\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2} \to$ پس $B=45^\circ$

③ الف)



$\sin 60^\circ=\frac{\sqrt{3}}{2}=\frac{50\sqrt{3}}{AB} \to AD=100 \to (50\sqrt{3})^2+x^2=(100)^2$

$7500+x^2=10000 \to x^2=2500$

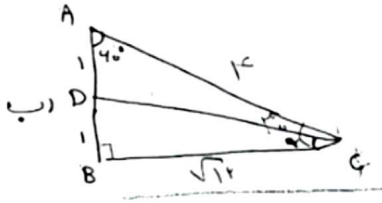
$BD=50$

$\sin 30^\circ=\frac{1}{2}=\frac{50\sqrt{3}}{AC} \to AC=100\sqrt{3}$

$(50\sqrt{3})^2+x^2=(100\sqrt{3})^2 \to 7500+x^2=30000 \to x=150=BC$

$CD=BC-BD=150-50=100$

ب)



$\sin 30^\circ=\frac{1}{2}=\frac{2}{AC} \to AC=4$

$2^2+BC^2=14 \to 14-4=12 \to BC=\sqrt{12}$

$\sin\alpha=\frac{1}{\sqrt{12}}$

$1+12=13 \to \sqrt{12}=CD$

⑤

$\frac{AB}{AC}=?$

$45+105=150 \to 180-150=30$

$\frac{AC}{\sin B}=\frac{CB}{\sin A}=\frac{AB}{\sin C} \to \frac{AC}{\frac{1}{2}}=\frac{BC}{\frac{\sqrt{2}}{2}}=\frac{AB}{\sin 105} \Rightarrow \frac{AB}{AC}=\frac{\sin 105}{\sin 60}=\frac{\sin 60\cos 45+\sin 45\cos 60}{\frac{\sqrt{3}}{2}}$

$\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}=\frac{\sqrt{6}+\sqrt{2}}{2}\cdot\frac{1}{2}$

⑥

$$\frac{S_{ABE}}{S_{BCD}} = \frac{\frac{1}{2}\times \sin\alpha \times 2\times 4}{\frac{1}{2}\times \sin\alpha \times 3\times 4} = \frac{8}{12} = \frac{2}{3}$$

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④

الف $3\times 4\times \sin 120^\circ = 12\times \frac{\sqrt{3}}{2} = 6\sqrt{3}$

ب $\frac{1}{2}\times 3\times 4\times \sin 120^\circ = \frac{1}{2}\times 12\times \frac{\sqrt{3}}{2} = 3\sqrt{3}$

$360 - 240 = 120 \div 2 = 60$

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⑦

$4^2 + 2^2 = 16 + 4 = 20$ $\quad \cos\theta = \frac{4}{\sqrt{20}}$

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⑧

$$\left.\begin{matrix} mx+b=0 \\ 2x+a=0 \end{matrix}\right\} \quad mx+b = 2x+a$$

$m=2, b=a \quad mb = 10$

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⑨

$f(x) = -\cos\pi + \cos\pi - \cot\pi = \frac{-\cos}{\sin}\pi$

$f\left(\frac{5\pi}{4}\right) = -\cot\frac{5\pi}{4} = -(-\sqrt{2}) = \sqrt{2}$

$\frac{5\pi}{4} \to 180^\circ$

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⑩

$$\frac{\sin\left(\frac{\pi}{2}+x\right)+\cos\left(\frac{\pi}{2}-x\right)}{\cos(2\pi+x)-\sin(4\pi-x)} = \frac{\cos\pi+\sin\pi}{\cos\pi-(-\sin\pi)} = \frac{\cos\pi+\sin\pi}{\sin\pi-\cos\pi}$$