سارا عزیزی - تکلیف شماره ۲۰

۱)

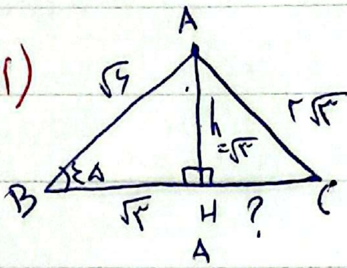
الف) $(\sin 135 + \sin 300)(\cos 315 - \cos 150) = \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{3}}{2}\right)\right)$

$= \frac{\sqrt{2}-\sqrt{3}}{2} \times \frac{\sqrt{2}+\sqrt{3}}{2} = \frac{2-3}{4} = -\frac{1}{4}$

ب) $\frac{1+3\tan^2 210}{4\cos^2 330 + 2\cos^2 225} = \frac{2}{3+1} = \frac{2}{4} = \frac{1}{2}$

۲)

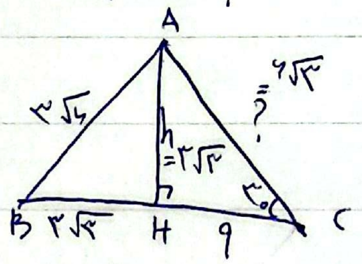
الف)



$h = \frac{\sqrt{2}}{2} \times \sqrt{6} = \sqrt{3} \rightarrow (\sqrt{12})^2 = (\sqrt{3})^2 + ?^2$

$\rightarrow ?^2 = 9 \rightarrow ? = 3$

ب)



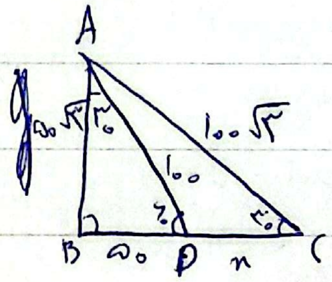
$\frac{\sqrt{3}}{2} \times ? = 9 \rightarrow ? = 6\sqrt{3}$

$h = \frac{6\sqrt{3}}{2} = 3\sqrt{3}$

$(3\sqrt{6})^2 = (3\sqrt{3})^2 + (BH)^2 \rightarrow BH = 3\sqrt{3} \rightarrow B = A = 45$

۳)

الف)



$50\sqrt{2} \times 2 = 100\sqrt{2} = AC$

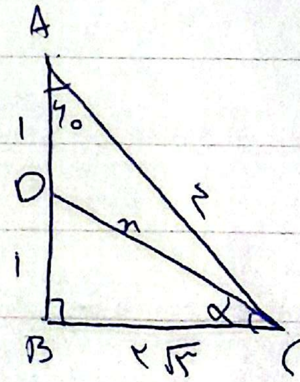
$50\sqrt{2} = AD \times \frac{\sqrt{2}}{2} \rightarrow AD = 100$

$\rightarrow BD = \frac{100}{2} = 50$

$\rightarrow (100\sqrt{3})^2 = (50\sqrt{3})^2 + BC^2 \rightarrow 30000 - 7500 = BC^2 = 22500 \rightarrow BC = 150$

$150 = BD + x = 50 + x \rightarrow x = 100$

ب)

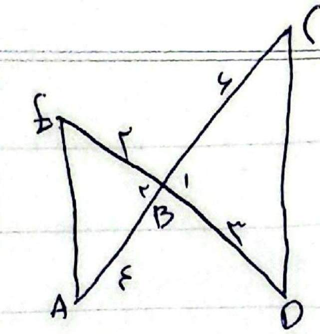


$AB = \frac{AC}{2} \rightarrow 2 = \frac{AC}{2} \rightarrow AC = 4$

$BC = \frac{\sqrt{3}}{2} \times 4 = 2\sqrt{3}$

$x^2 = 1^2 + (2\sqrt{3})^2 = 1 + 12 = 13 \rightarrow x = \sqrt{13}$

$\rightarrow \sin\alpha = \frac{\text{مقابل}}{\text{وتر}} = \frac{1}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}} = \frac{\sqrt{13}}{13}$

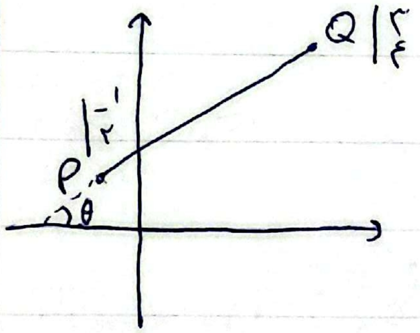


$$\frac{S_{\Delta ABE}}{S_{\Delta BCD}} = \frac{\frac{1}{2}\times 2\times 4\times \sin B_2}{\frac{1}{2}\times 3\times 6\times \sin B_1} \xrightarrow{B_1=B_2} \sin B_1 = \sin B_2$$ ۵

$$\rightarrow \frac{S_{ABE}}{S_{BCD}} = \frac{4\times \cancel{\sin B_2}}{3\times 3\times \cancel{\sin B_1}} = \frac{4}{9}$$

الف) $\cancel{2}\times\frac{1}{\cancel{2}}\times 3\times \cancel{4}\times \sin 120$ $\left(\sin 120 = \frac{\sqrt{3}}{2}\right) = 6\sqrt{3}$ ۶

ب) $\frac{1}{\cancel{2}}\times\cancel{4}\times 3\times \frac{\sqrt{3}}{\cancel{2}} = 3\sqrt{3}$ ($\uparrow \sin(120)$)

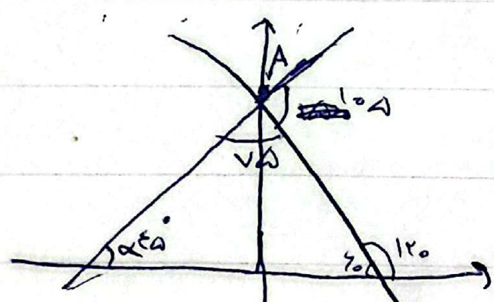


$Q\,|\,{3 \atop 4}$ $\quad P\,|\,{-1 \atop 2}$

$$\tan\theta = a = \frac{\Delta y}{\Delta x} = \frac{4-2}{3-(-1)} = \frac{1}{2}$$ ۷

$$\rightarrow 1+\tan^2\theta = \frac{1}{\cos^2\alpha} \rightarrow 1+\frac{1}{4} = \frac{5}{4} = \frac{1}{\cos^2}$$

$$\rightarrow \cos^2 = \frac{4}{5} \rightarrow \cos = \frac{2}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$



$y = mx + b$

$$\tan 120 = -\sqrt{3} = m$$ ۸

$A) \rightarrow y = x + 2/5 \xrightarrow{x=0} y = 0 + 2/5 = 2/5$

$\rightarrow y = -\sqrt{3}x + b \xrightarrow[x=0]{y=2/5} 2/5 = (-\sqrt{3}\times 0) + b$

$\rightarrow b = 2/5$

$2y = 2x + 5$

$\downarrow$

$y = x + 2/5$

$\downarrow$

$a = 1 = \tan\alpha \rightarrow \alpha = 45$

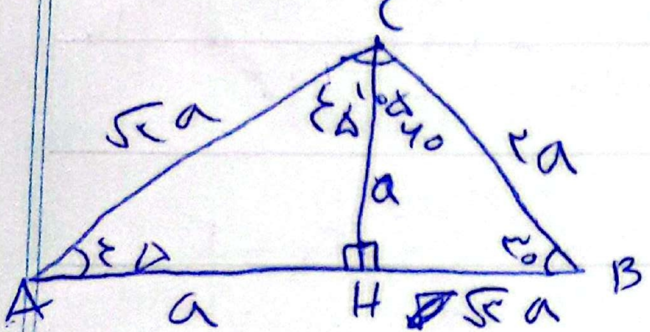
$$\Rightarrow mb = -\sqrt{3}\times \frac{5}{2} = -\frac{5\sqrt{3}}{2}$$

~~$\rightarrow f\left(\frac{5\pi}{6}\right) = \cos\left(\pi - \frac{5\pi}{6}\right) + \sin\left(\frac{\pi}{2} - \frac{5\pi}{6}\right) - \tan\left(\frac{3\pi}{2} + \frac{5\pi}{6}\right)$~~ ۹

$$\rightarrow f(x) = -\cos x + \cos x + \cot x = \cot x \rightarrow f\left(\frac{5\pi}{6}\right) = \cot\frac{5\pi}{6}$$

$$= \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$$

۴



$BC = a \times 2 = 2a$

$BH = \frac{\sqrt{3}}{\cancel{2}} \times \cancel{2}a = \sqrt{3}\,a$

$a^2 + a^2 = 2a^2 = AC^2 \rightarrow AC = \sqrt{2}\,a$

$\frac{AB}{AC} = \frac{a+\sqrt{3}a}{\sqrt{2}a} = \frac{\cancel{a}(1+\sqrt{3})}{\sqrt{2}\cancel{a}} = \frac{1+\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{2}+\sqrt{6}}{2}$

۱۰

$$\frac{\sin\left(\frac{16\pi}{2} + \frac{\pi}{2} + x\right) + \cos\left(\frac{1\cancel{4}^{\,7}\pi}{\cancel{4}} + \frac{\pi}{2} - x\right)}{\cos(3\pi + x) - \sin(6\pi - x)}$$

$$= \frac{\sin\left(\frac{\pi}{2} + x\right) + \cos\left(\pi + \frac{\pi}{2} - x\right)}{\cos(\pi + x) - \sin(2\pi - x)} = \frac{\cos x - \sin x}{-\cos x + (-\sin x)} = \frac{\cos x - \sin x}{-(\cos x - \sin x)}$$

$= -1$