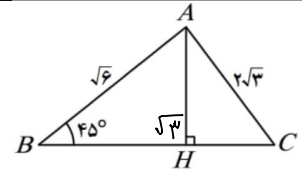


(الف) $(\sin 135^\circ + \sin 30^\circ)(\cos 315^\circ - \cos 15^\circ) = \left(\frac{\sqrt{2}}{2} + (-\frac{\sqrt{2}}{2})\right) \left(\frac{\sqrt{2}}{2} - (-\frac{\sqrt{2}}{2})\right) = \left(\frac{\sqrt{2}}{2}\right)^2 - \left(-\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} - \frac{2}{4} = \frac{1}{2} - \frac{1}{2} = 0$ (1)

ب) $\frac{1 + 3 \tan^2 21^\circ}{4 \cos^2 33^\circ + 2 \cos^2 22^\circ} = \frac{1 + 3 \left(\frac{\sqrt{2}}{2}\right)^2}{4 \left(\frac{\sqrt{2}}{2}\right)^2 + 2 \left(-\frac{\sqrt{2}}{2}\right)^2} = \frac{1 + 3 \times \frac{2}{4}}{4 \times \frac{2}{4} + 2 \times \frac{2}{4}} = \frac{1 + \frac{3}{2}}{2 + 1} = \frac{\frac{5}{2}}{3} = \frac{5}{6}$

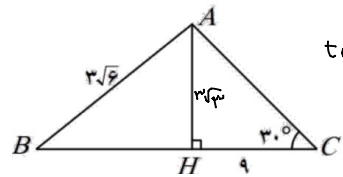


$\sin 45^\circ = \frac{\sqrt{2}}{2} = \frac{AH}{\sqrt{6}} \rightarrow \sqrt{2} = 2AH \rightarrow AH = \frac{\sqrt{2}}{2} = \sqrt{3}$

$(\sqrt{3})^2 + HC^2 = (2\sqrt{3})^2 \rightarrow 3 + HC^2 = 12 \rightarrow HC^2 = 9 \rightarrow HC = 3$

(2)

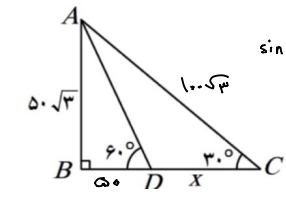
(الف)



$\tan 30^\circ = \frac{AH}{\frac{BC}{2}} = \frac{\sqrt{3}}{2} \rightarrow AH = \frac{3\sqrt{3}}{2} = 3\sqrt{3}$

$\sin B = \frac{3\sqrt{3}}{3\sqrt{6}} = \frac{\sqrt{3}}{\sqrt{6}} = \frac{\sqrt{2}}{2} \rightarrow 45^\circ$

(ب)



$\sin 60^\circ = \frac{5\sqrt{3}}{10} = \frac{1}{2} \rightarrow AC = 10\sqrt{3}$

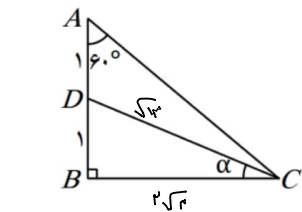
$\tan 45^\circ = \frac{5\sqrt{3}}{BD} = \sqrt{3} \rightarrow BD = 5$

$(5\sqrt{3})^2 + BC^2 = (10\sqrt{3})^2$

$75 + BC^2 = 300 \rightarrow BC^2 = 225 \rightarrow BC = 15$

(3)

(الف)

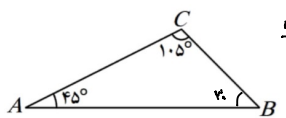


$\tan 45^\circ = \sqrt{3} = \frac{BC}{1} \rightarrow BC = 2\sqrt{3}$

$1 + (2\sqrt{3})^2 = 13 \rightarrow DC^2 = 12 \rightarrow DC = 2\sqrt{3}$

$\sin \alpha = \frac{1}{\sqrt{13}} \rightarrow \frac{\sqrt{13}}{13}$

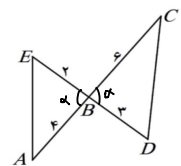
(ب)



$\frac{BC}{\sqrt{2}} = \frac{AC}{1} = \frac{AB}{\frac{1}{\sqrt{2}}} \rightarrow \frac{BC}{\sqrt{2}} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2} \rightarrow BC = 2$

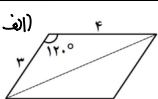
$\sin 60^\circ = \sin(45^\circ + 15^\circ) = \sin 45^\circ \cos 15^\circ + \cos 45^\circ \sin 15^\circ = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} + \frac{1}{2} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{4} = \frac{3\sqrt{2}}{4}$

(4)

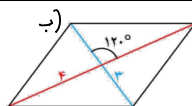


$\frac{S_{ABE}}{S_{ACD}} = \frac{\frac{1}{2} \times AB \times AC \times \sin 45^\circ}{\frac{1}{2} \times AC \times CD \times \sin 30^\circ} = \frac{1}{2}$

(5)

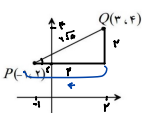


$S = \frac{1}{2} \times AB \times AC \times \sin 45^\circ = \frac{1}{2} \times 1 \times \sqrt{2} \times \frac{\sqrt{2}}{2} = \frac{1}{2}$



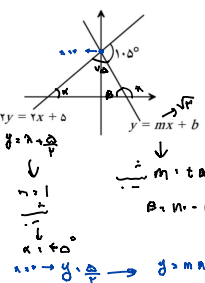
$S = \frac{1}{2} \times AB \times AC \times \sin 45^\circ = \frac{1}{2} \times 1 \times \sqrt{2} \times \frac{\sqrt{2}}{2} = \frac{1}{2}$

(6)



$\cos \theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \rightarrow \theta = 45^\circ$

(7)



$n, b = \frac{1}{\sqrt{2}} \times \sqrt{2} = 1$

(8)

$$f(x) = \cos(\pi + x) + \sin\left(\frac{\pi}{3} - x\right) - \tan\left(\frac{2\pi}{3} + x\right) \rightarrow f(\lambda) = -\cos \lambda + \sin(\lambda) - \cot(\lambda) = \cot(\lambda) \rightarrow -\cot\left(\frac{2\pi}{3}\right) = +\sqrt{3} \quad (9)$$

$$\frac{\sin\left(\frac{14\pi}{3} + x\right) + \cos\left(\frac{16\pi}{3} - x\right)}{\cos\left(\frac{2\pi}{3} + x\right) - \sin\left(\frac{4\pi}{3} - x\right)} = \frac{\cos(\lambda) - \sin(\lambda)}{-\cos \lambda - (-\sin \lambda)} = \frac{\cos \lambda - \sin \lambda}{-(\cos \lambda - \sin \lambda)} = -1 \quad (10)$$

$$\left(\frac{14\pi}{3} + x\right), \left(\frac{16\pi}{3} - x\right) \rightarrow \left(\sin \frac{\pi}{3} + \lambda\right) = \cos \lambda$$