

$$(1! + 3! + \dots + 25!) (2! + 4! + \dots + 24!) \\ \downarrow \qquad \qquad \qquad \downarrow \\ 1! + 3! = 4 \qquad \qquad \qquad 2! + 4! = 2 + 24 = 26$$

$$4 \times 26 = 104 \rightarrow \text{مجموع}$$

$$\text{الف) } \sqrt[4]{(4+\sqrt{5})^{-1}} \times \sqrt{1+\sqrt{5}} = \sqrt[4]{(4+\sqrt{5})^{-1}} \times \sqrt[4]{(1+\sqrt{5})^2} = \sqrt[4]{\frac{(1+\sqrt{5})^2}{4+\sqrt{5}}}$$

$$= \sqrt[4]{\frac{1+2\sqrt{5}+5}{4+\sqrt{5}}} = \sqrt[4]{\frac{6+2\sqrt{5}}{4+\sqrt{5}}} = \sqrt[4]{\frac{2(3+\sqrt{5})}{4+\sqrt{5}}} = \sqrt[4]{2}$$

$$\text{ب) } \left( \frac{\sqrt{2+\sqrt{5}}}{\sqrt{10}+2} \right) (\sqrt{3-\sqrt{5}} - \sqrt{3+\sqrt{5}}) = \left( \frac{\sqrt{2+\sqrt{5}}}{\sqrt{2}(\sqrt{5}+\sqrt{2})} \right) (\sqrt{3-\sqrt{5}} - \sqrt{3+\sqrt{5}})$$

$$\Rightarrow \frac{\sqrt{3-\sqrt{5}} - \sqrt{3+\sqrt{5}}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{4-2\sqrt{5}} - \sqrt{4+2\sqrt{5}}}{2} = \frac{(\sqrt{5}-1)^2 - (\sqrt{5}+1)^2}{2}$$

$$\frac{\sqrt{5}-1 - \sqrt{5}-1}{2} = -1$$



$$(1! + 3! + \dots + 25!) (2! + 4! + \dots + 24!) \quad (1)$$

$\downarrow$   $\downarrow$   
 $1! + 3! = 4$   $2! + 4! = 2 + 24 = 26$

$4 \times 26 = 104$  ۱۰۴  $\rightarrow$  تم یکان

الف)  $\sqrt[4]{(4+\sqrt{7})^{-1}} \times \sqrt{1+\sqrt{7}} = \sqrt[4]{(4+\sqrt{7})^{-1}} \times \sqrt[4]{(1+\sqrt{7})^2} = \sqrt[4]{\frac{(1+\sqrt{7})^2}{4+\sqrt{7}}}$  (۲)

$= \sqrt[4]{\frac{1+2\sqrt{7}+7}{4+\sqrt{7}}} = \sqrt[4]{\frac{8+2\sqrt{7}}{4+\sqrt{7}}} = \sqrt[4]{\frac{2(4+\sqrt{7})}{4+\sqrt{7}}} = \sqrt[4]{2}$  ۴

ب)  $\left( \frac{\sqrt{2}+\sqrt{5}}{\sqrt{10}+2} \right) (\sqrt{3}-\sqrt{5} - \sqrt{3+\sqrt{5}}) = \left( \frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}(\sqrt{5}+\sqrt{2})} \right) (\sqrt{3}-\sqrt{5} - \sqrt{3+\sqrt{5}})$

$\downarrow$   
 $\sqrt{2} \times \sqrt{5}$

کریانه  $\Rightarrow \frac{\sqrt{3}-\sqrt{5} - \sqrt{3+\sqrt{5}}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{4-2\sqrt{5}} - \sqrt{4+2\sqrt{5}}}{2} = \frac{(\sqrt{5}-1)^2 - (\sqrt{5}+1)^2}{2}$

$\frac{\sqrt{5}-1 - \sqrt{5}-1}{2} = -1$  -۱



$$\text{الف) } \frac{\sqrt{1} + \sqrt{27}}{5 - \sqrt{4}} - 2(\sqrt[3]{9} - 1)^{-1} = \frac{2\sqrt{2} + 3\sqrt{3}}{5 - \sqrt{4}} - \frac{2}{\sqrt{3} - 1} \quad (1)$$

$$\Rightarrow \frac{(2\sqrt{2} + 3\sqrt{3})(\sqrt{2} + \sqrt{3})}{((\sqrt{2})^2 + (\sqrt{3})^2 - \sqrt{3} \times \sqrt{2})(\sqrt{2} + \sqrt{3})} - \frac{2(\sqrt{3} + 1)}{\sqrt{3} - 1} = \sqrt{2} + \sqrt{3} - \sqrt{3} - 1 = \sqrt{2} - 1$$

$\xrightarrow{\quad} 2\sqrt{2} + 3\sqrt{3}$

$$\text{ب) } \frac{\sqrt{27} - 1}{2 + \sqrt{3}} + (2 - \sqrt{3})^{-1} = \left( \frac{3\sqrt{3} - 1}{2 + \sqrt{3}} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1} \right) + \left( \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} \right)$$

$$= \frac{3\sqrt{3} - 1(\sqrt{3} - 1)}{2\sqrt{3} - 1} + \frac{2 + \sqrt{3}}{1} = \sqrt{3} - 1 + 2 + \sqrt{3} = 2\sqrt{3} + 1$$

$$\text{الف) } \frac{\sqrt{1 + \sqrt{3}} + \sqrt{\sqrt{3} - 1}}{\sqrt{\sqrt{3} - \sqrt{2}}} - 2 \Rightarrow \frac{\sqrt{1 + \sqrt{3}} + \sqrt{\sqrt{3} - 1}}{\sqrt{\sqrt{3} - \sqrt{2}}} \times \frac{\sqrt{\sqrt{2} + \sqrt{2}}}{\sqrt{\sqrt{2} + \sqrt{2}}} = \quad (1)$$

$$\left( \sqrt{1 + \sqrt{3}} + \sqrt{\sqrt{3} - 1} \right) \left( \sqrt{\sqrt{2} - \sqrt{2}} \right) = \sqrt{[(\sqrt{1 + \sqrt{3}} + \sqrt{\sqrt{3} - 1})(\sqrt{\sqrt{2} - \sqrt{2}})]^2}$$

$$= \sqrt{(2\sqrt{2} - 2\sqrt{2})(\sqrt{2} - \sqrt{2})} = (\sqrt{3} - \sqrt{2})\sqrt{2} \Rightarrow \sqrt{2}(\sqrt{3} - \sqrt{2}) - 2 = \sqrt{4} - 2$$

$$\text{ب) } \sqrt[3]{\sqrt[3]{27} \cdot 1} \times \sqrt[3]{142} \times \sqrt[3]{2\sqrt{2}} = 2^{\frac{1}{3}} \times 3 \times 2^{\frac{1}{6}} \times 2 \times 2^{\frac{1}{6}} = 3 \times 2 = 6$$

$$\frac{2^0 + 2^{n+1} + 2^{n+2} + 2^{n+3} + 2^{n+4} + 2^{n+5} + 2^{n+6}}{2^{n-2} + 2^{n-1} + 2^n + 2^{n+1} + 2^{n+2} + 2^{n+3} + 2^{n+4}} = 2^2$$

$$\frac{2^n(1 + 2 + 2^2 + \dots + 2^6)}{2^{n-2}(1 + 2 + 2^2 + \dots + 2^6)} = \frac{2^n(1 - 2^7)}{2^{n-2}(1 - 2^7)} = \frac{2^{n-2}}{2^{n-2}} = 1 \rightarrow n = 2$$

$$a = \sqrt[4]{\sqrt{4}-2}$$

$$b = \sqrt[4]{\sqrt{4}+2}$$

$$(a^2 + b^2 + ab)^2 (a^2 + b^2 - ab)^2 = t(2 - \sqrt{4}) \quad (4)$$

$$\Rightarrow ((a^2 + b^2)^2 - (ab)^2)^2 = (a^4 + b^4 + 2ab^2 - 2a^2b)^2$$

$$= ((\sqrt[4]{4-2})^2 + (\sqrt[4]{4+2})^2 + 2\sqrt[4]{(4-2)(4+2)})^2$$

$$= \sqrt{4-2} + \sqrt{4+2} + 2(\sqrt{4-2})$$

$$= 2\sqrt{4} - 2\sqrt{2} \xrightarrow{\text{r.h.s.}} (2\sqrt{4} - 2\sqrt{2})^2 =$$

$$2^2 + 16 - 16\sqrt{2} = \frac{2^2 - 14\sqrt{2}}{2}$$

$$* \quad 22 - 14\sqrt{2} = t(2 - \sqrt{4})$$

$$14(2 - \sqrt{4}) = t(2 - \sqrt{4})$$

$$t = 14$$

$$a = \sqrt[4]{2 - \varepsilon\sqrt{2}}$$

$$= \sqrt[4]{(2 - \sqrt{2})^2} \Rightarrow$$

$$\boxed{\sqrt{2 - \sqrt{2}} = a}$$

$$(a + \frac{1}{a})^2 (a + \frac{1}{a} - \sqrt{2})^2 = t^2 \quad (v)$$

$$\downarrow$$

$$((a + \frac{1}{a})^2 - 2)^2 = (a^2 + \frac{1}{a^2} + 2 - 2)^2 =$$

$$(2 - \sqrt{2} + \frac{1}{2 - \sqrt{2}})^2 = (2 - \sqrt{2} + 2 + \sqrt{2})^2$$

$$= 2^2 = 14 \Rightarrow t^2 = 2^2 \Rightarrow t = 2$$

$$A = \sqrt[4]{2\sqrt[3]{14}} \left(\frac{1}{2}\right)^{-\frac{2}{3}} = 2^{\frac{2}{3}} \times 2^{\frac{2}{3}} = 2^{\frac{4}{3}} = 2 \quad (1)$$

$$(2A)^{\frac{1}{2}} = (2^2)^{\frac{1}{2}} = \left(\frac{1}{2^2}\right)^{\frac{1}{2}} = \sqrt{\frac{1}{2^2}} = \frac{1}{2}$$

$$\frac{\sqrt[4]{a}}{\sqrt[4]{a^2}} = 2^2 \Rightarrow \sqrt[4]{\frac{1}{a^2}} = 2^2 \Rightarrow \frac{1}{a^2} = 2^4 \Rightarrow a = \frac{1}{2^2 \times \sqrt{2}}$$

$$\boxed{a = \frac{\sqrt{2}}{4}}$$

$$\frac{\frac{1}{a} - 2}{1 + \sqrt{2}} = \frac{\frac{4}{\sqrt{2}} - 2}{1 + \sqrt{2}} = \frac{\frac{4 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} - 2}{1 + \sqrt{2}} = \frac{\frac{4\sqrt{2}}{2} - 2}{1 + \sqrt{2}} = \frac{2\sqrt{2} - 2}{1 + \sqrt{2}}$$

$$\frac{2(\sqrt{2} - 1)}{\sqrt{2} + 1} \times \frac{(\sqrt{2} - 1)}{\sqrt{2} - 1} = \frac{2(\sqrt{2} - 1)^2}{2} = \frac{2(2 + 1 - 2\sqrt{2})}{2} = \frac{3 - 2\sqrt{2}}{1} =$$

$$\frac{3 - 2\sqrt{2}}{1}$$



$$\sqrt{n+a} - \sqrt{n-\varepsilon} = r \Rightarrow (\sqrt{n+a} - \sqrt{n-\varepsilon})(\sqrt{n+a} + \sqrt{n-\varepsilon}) \quad (1)$$

$$\cancel{n+a} - \cancel{n} + \varepsilon = r \sqrt{n+a} + \sqrt{n-\varepsilon} \Rightarrow \frac{a+\varepsilon}{r} = \sqrt{n+a} + \sqrt{n-\varepsilon}$$

$$\sqrt{n+a} + \cancel{\sqrt{n-\varepsilon}} - r = \frac{a+\varepsilon-\varepsilon}{r} = \frac{a}{r}$$