نام و نام خانوادگی: فاطمه زهرا جمال زاده   کلاس: دهم دختر A   شماره تکلیف: ۲۱

۱)

$$(1! + 3! + \dots + 25!)\,(2! + 4! + \dots + 26!)$$

$$1! + 3! = 7 \qquad 2! + 4! = 2 + 24 = 26$$

$$7 \times 6 = 4\boxed{2} \longrightarrow \text{رقم یکان}$$

۲) الف)

$$\sqrt[4]{(4+\sqrt{7})^{-1}} \times \sqrt{1+\sqrt{7}} = \sqrt[4]{(4+\sqrt{7})^{-1}} \times \sqrt[4]{(1+\sqrt{7})^{2}} = \sqrt[4]{\frac{(1+\sqrt{7})^{2}}{4+\sqrt{7}}}$$

$$= \sqrt[4]{\frac{1+2\sqrt{7}+7}{4+\sqrt{7}}} = \sqrt[4]{\frac{8+2\sqrt{7}}{4+\sqrt{7}}} = \sqrt[4]{\frac{2\cancel{(4+\sqrt{7})}}{\cancel{4+\sqrt{7}}}} = \boxed{\sqrt[4]{2}}$$

ب)

$$\left(\frac{\sqrt{2}+\sqrt{5}}{\underbrace{\sqrt{10}}_{\sqrt{2}\times\sqrt{5}}+2}\right)\left(\sqrt{3-\sqrt{5}} - \sqrt{3+\sqrt{5}}\right) = \left(\frac{\cancel{\sqrt{2}+\sqrt{5}}}{\sqrt{2}\,\cancel{(\sqrt{5}+\sqrt{2})}}\right)\left(\sqrt{3-\sqrt{5}} - \sqrt{3+\sqrt{5}}\right)$$

$$\xrightarrow{\text{گویا کردن}} \frac{\sqrt{3-\sqrt{5}} - \sqrt{3+\sqrt{5}}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6-2\sqrt{5}} - \sqrt{6+2\sqrt{5}}}{2} = \frac{\sqrt{(\sqrt{5}-1)^2} - \sqrt{(\sqrt{5}+1)^2}}{2}$$

$$\frac{\cancel{\sqrt{5}} - 1 - \cancel{\sqrt{5}} - 1}{2} = \boxed{-1}$$

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۱)

$$(1!+3!+\cdots+25!)(2!+4!+\cdots+26!)$$

$$1!+3!=7 \qquad 2!+4!=2+24=26$$

$$7\times 6 = 4\boxed{2} \longrightarrow$$ رقم یکان

۲) الف)

$$\sqrt[4]{(4+\sqrt7)^{-1}} \times \sqrt{1+\sqrt7} = \sqrt[4]{(4+\sqrt7)^{-1}} \times \sqrt[4]{(1+\sqrt7)^2} = \sqrt[4]{\frac{(1+\sqrt7)^2}{4+\sqrt7}}$$

$$= \sqrt[4]{\frac{1+2\sqrt7+7}{4+\sqrt7}} = \sqrt[4]{\frac{8+2\sqrt7}{4+\sqrt7}} = \sqrt[4]{\frac{2(\cancel{4+\sqrt7})}{\cancel{4+\sqrt7}}} = \boxed{\sqrt[4]{2}}$$

ب)

$$\left(\frac{\sqrt2+\sqrt5}{\underbrace{\sqrt{10}}_{\sqrt2\times\sqrt5}+2}\right)\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right) = \left(\frac{\cancel{\sqrt2+\sqrt5}}{\sqrt2(\cancel{\sqrt5+\sqrt2})}\right)\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)$$

گویا کردن

$$\Longrightarrow \frac{\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}}{\sqrt2}\times\frac{\sqrt2}{\sqrt2} = \frac{\sqrt{6-2\sqrt5}-\sqrt{6+2\sqrt5}}{2} = \frac{\sqrt{(\sqrt5-1)^2}-\sqrt{(\sqrt5+1)^2}}{2}$$

$$\frac{\cancel{\sqrt5}-1-\cancel{\sqrt5}-1}{2} = \boxed{-1}$$

(۳ الف)

$$\frac{\sqrt{8}+\sqrt{27}}{5-\sqrt{6}}-2\left(\sqrt[4]{3^2}-1\right)^{-1}=\frac{2\sqrt{2}+3\sqrt{3}}{\underbrace{5}_{(\sqrt2)^2+(\sqrt3)^2}-\underbrace{\sqrt{6}}_{\sqrt3\times\sqrt2}}-\frac{2}{\sqrt{3}-1}$$

مثل فرمول

$$\Rightarrow\frac{(2\sqrt{2}+\cancel{3\sqrt{3}})(\sqrt{2}+\sqrt{3})}{\underbrace{\left((\sqrt{2})^2+(\sqrt{3})^2-\sqrt{3}\times\sqrt{2}\right)(\sqrt{2}+\sqrt{3})}_{2\sqrt{2}+\cancel{3\sqrt{3}}}}-\frac{\cancel{2}(\sqrt{3}+1)}{\frac{3-1}{\cancel{2}}}=\sqrt{2}+\sqrt{3}-\sqrt{3}-1=\sqrt{2}-1$$

ب)

$$\frac{\sqrt{27}-1}{4+\sqrt{3}}+(2-\sqrt{3})^{-1}=\left(\frac{3\sqrt{3}-1}{3+\underbrace{1-\sqrt{3}}}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}\right)+\left(\frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}\right)$$

$$=\frac{\cancel{3\sqrt{3}-1}(\sqrt{3}-1)}{\cancel{3\sqrt{3}-1}}+\frac{2+\sqrt{3}}{1}=\sqrt{3}-1+2+\sqrt{3}=2\sqrt{3}+1$$

(۴ الف)

$$\frac{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}}-2\rightsquigarrow\frac{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}}\times\frac{\sqrt{\sqrt{3}+\sqrt{2}}}{\sqrt{\sqrt{3}+\sqrt{2}}}=$$

$$\left(\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}\right)\left(\sqrt{\sqrt{3}-\sqrt{2}}\right)=\sqrt{\left[\left(\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}\right)\left(\sqrt{\sqrt{3}-\sqrt{2}}\right)\right]^2}$$

$$=\sqrt{(2\sqrt{3}-2\sqrt{2})(\sqrt{3}-\sqrt{2})}=(\sqrt{3}-\sqrt{2})\sqrt{2}\Rightarrow\sqrt{2}(\sqrt{3}-\sqrt{2})-2=\sqrt{6}-4$$

ب)

$$\underbrace{\sqrt[4]{\sqrt[3]{2^8}}}_{\sqrt[12]{2^8}=\sqrt[3]{2^2}}\times\underbrace{\sqrt[4]{162}}_{\sqrt[4]{3^4\times2}}\times\underbrace{\sqrt{4\sqrt[6]{2}}}_{\sqrt[12]{2^{12}\times2}}=2^{\frac{2}{3}}\times3\times2^{\frac{1}{4}}\times2\times2^{\frac{1}{12}}=3\times4=12$$

(۵

$$\frac{3^x+3^{x+1}+3^{x+2}+3^{x+3}+3^{x+4}+3^{x+5}}{2^{x-2}+2^{x-1}+2^x+2^{x+1}+2^{x+2}+2^{x+3}}=52$$

28×26

$$\frac{3^x(1+3+9+\dots+3^5)}{2^{x-2}(1+2+4+\dots 2^5)}=\frac{3^x\left(\frac{3^6-1}{2}\right)}{2^{x-2}\left(\frac{2^6-1}{1}\right)}=\frac{3^{x-2}}{2^{x-2}}=1\to x=2$$

63

(6

$$a=\sqrt[4]{\sqrt{6}-2}$$
$$b=\sqrt[4]{\sqrt{6}+2}$$

$$(a^2+b^2-2ba)^2(a^2+b^2+2ab)^2=t(2-\sqrt{3})$$

$$\Rightarrow \big((a^2+b^2)^2-4a^2b^2\big)^2=\big(a^4+b^4+2a^2b^2\cancel{-4a^2b^2}-2a^2b^2\big)^2$$

$$=\left(\left(\sqrt[4]{\sqrt{6}-2}\right)^4+\left(\sqrt[4]{\sqrt{6}+2}\right)^4-2\sqrt{(\sqrt{6}-2)(\sqrt{6}+2)}\right)$$

$$=\sqrt{6}\cancel{-2}+\sqrt{6}\cancel{+2}-2\left(\sqrt{6-4}\right)$$

$$=2\sqrt{6}-2\sqrt{2}\xrightarrow{\text{توان ۲}}(2\sqrt{6}-2\sqrt{2})^2=$$

$$24+8-8\sqrt{12}=32-16\sqrt{3}$$

$$*\quad 32-16\sqrt{3}=t(2-\sqrt{3})$$
$$16(2-\cancel{\sqrt{3}})=t(2-\cancel{\sqrt{3}})$$

$$t=16$$

(7

$$a=\sqrt[4]{7-4\sqrt{3}}$$
$$=\sqrt[4]{(2-\sqrt{3})^2}\Rightarrow$$
$$\boxed{\sqrt{2-\sqrt{3}}=a}$$

$$\left(a+\frac{1}{a}+\sqrt{2}\right)^2\left(a+\frac{1}{a}-\sqrt{2}\right)^2=2^t$$

$$\left(\left(a+\frac{1}{a}\right)^2-2\right)^2=\left(a^2+\frac{1}{a^2}+\cancel{2-2}\right)^2=$$

$$\left(2-\sqrt{3}+\frac{1}{2-\sqrt{3}}\right)^2=\left(2-\sqrt{3}+2+\sqrt{3}\right)^2$$

$$=4^2=16\rightarrow 2^t=2^4\Rightarrow t=4$$

(8

$$A=\sqrt[5]{4\sqrt[3]{16}}\left(\frac{1}{2}\right)^{-\frac{4}{3}}=2^{\frac{2}{3}}\times 2^{\frac{4}{3}}=2^{\frac{6}{3}}=\boxed{4}$$

$$(2A)^{-\frac{1}{3}}=(2^3)^{-\frac{1}{3}}=\left(\frac{1}{2^3}\right)^{\frac{1}{3}}=\sqrt[3]{\frac{1}{2^3}}=\frac{1}{2}$$

(9

$$\frac{\sqrt[7]{a}}{\sqrt[7]{a^{15}}}=27\rightarrow\sqrt[7]{\frac{1}{a^{14}}}=27\rightarrow\frac{1}{a^2}=27\Rightarrow a=\frac{1}{3\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$\boxed{a=\frac{\sqrt{3}}{9}}$$

$$\frac{\frac{1}{a}-3}{1+\sqrt{3}}=\frac{\frac{9}{\sqrt{3}}-3}{1+\sqrt{3}}=\frac{\frac{9\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}-3}{1+\sqrt{3}}=\frac{\frac{9\sqrt{3}}{3}-\frac{9}{3}=\frac{\cancel{9}^{3}(\sqrt{3}-1)}{\cancel{3}}}{1+\sqrt{3}}$$

$$\frac{3(\sqrt{3}-1)}{\sqrt{3}+1}\times\frac{(\sqrt{3}-1)}{\sqrt{3}-1}=\frac{3(\sqrt{3}-1)^2}{2}=\frac{3(3+1-2\sqrt{3})}{2}=\frac{12-6\sqrt{3}}{2}=$$

$$6-3\sqrt{3}$$

$$\sqrt{x+a} - \sqrt{x-۴} = ۲ \Rightarrow \left(\sqrt{x+a} - \sqrt{x-۴}\right)\left(\sqrt{x+a} + \sqrt{x-۴}\right) \quad (۱۰$$

$$\cancel{x}+a - \cancel{x}+۴ = ۲\left(\sqrt{a+x} + \sqrt{x-۴}\right) \Rightarrow \frac{a+۴}{۲} = \sqrt{a+x} + \sqrt{x-۴}$$

$$\underbrace{\sqrt{x+a} + \sqrt{x-۴}}_{\frac{a+۴}{۲}} - ۲ = \frac{a+۴-۴}{۲} = \boxed{\frac{a}{۲}}$$