(۱)

$(1!+3!+5!+\cdots+25!)(2!+4!+6!+\cdots+26!)$

از !۵ به بعد رقم یکان همگی صفر خواهد شد پس. برای پرانتز اول داریم:

$(1!+3!)=1+6=7$

و برای پرانتز دوم هم داریم:

$(2!+4!)=2+24=26 \Longrightarrow 6\times 7=42$

در نتیجه رقم یکان ۲ می‌باشد

---

(۲)

الف) $\sqrt[4]{(4+\sqrt7)^{-1}}\times\sqrt{1+\sqrt7}$, $A=\frac{1}{(4+\sqrt7)^{\frac14}}\times(1+\sqrt7)^{\frac12}$, $4+\sqrt7=\frac{(1+\sqrt7)^2}{2}$

$\left(\frac{(1+\sqrt7)^2}{2}\right)^{\frac14}=\frac{(1+\sqrt7)^{\frac12}}{2^{\frac14}} \Rightarrow A=\frac{1}{\frac{(1+\sqrt7)^{\frac12}}{2^{\frac14}}}\times(1+\sqrt7)^{\frac12}=\frac{2^{\frac14}}{(1+\sqrt7)^{\frac12}}\times(1+\sqrt7)^{\frac12}=A=\boxed{2^{\frac14}=\sqrt[4]{2}}$

ب) $\left(\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\right)\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right) \Rightarrow \frac{\sqrt2}{2}\times\left((3-\sqrt5)^{\frac12}-(3+\sqrt5)^{\frac12}\right)$

$\frac{\sqrt2+\sqrt5\,(\sqrt{10}-2)}{\sqrt{10}+2\times(\sqrt{10}-2)} \Rightarrow \frac{2\sqrt5-2\sqrt2+5\sqrt2-2\sqrt5}{6}=\frac{\sqrt2}{2}$; $3-\sqrt5=\frac{(\sqrt5-1)^2}{2}$, $3+\sqrt5=\frac{(\sqrt5+1)^2}{2}$ $\xrightarrow{\text{جایگذاری}}$ جواب $\boxed{A=\frac{\sqrt{10}-\sqrt2-2\sqrt5-2}{2\sqrt2}}$

---

(۳)

الف) $\frac{\sqrt8+\sqrt{27}\,(5+\sqrt6)}{5-\sqrt6\,(5+\sqrt6)}-2\left(\sqrt[4]{9}-1\right)^{-1}=\frac{5\sqrt{2^3}+5\sqrt{3^3}+\sqrt{3\times2^4}+\sqrt{3^4\times2}-19\sqrt3-19}{19}=$; $\sqrt[4]{3^2}=\sqrt3$

$\frac{(\sqrt{2^3}+\sqrt{3^3})(5\times\sqrt6)}{25-6}$; $-2\times\frac{1(\sqrt3+1)}{(\sqrt3-1)(\sqrt3+1)}=\frac{(\sqrt3+1)\times19}{1\times19}=\boxed{19\sqrt3+19}$; $\frac{19\sqrt3-19\sqrt3+19\sqrt2-19}{19}=\frac{19(\sqrt2-1)}{19}=\boxed{\sqrt2-1}$ جواب

ب) $\frac{\sqrt{27}-1}{4+\sqrt3}+(2-\sqrt3)^{-1}$

$\frac{\sqrt{3^3}-1}{4+\sqrt3}\times\frac{(4-\sqrt3)}{(4-\sqrt3)}=\frac{12\sqrt3-4-9+\sqrt3}{4-3}=13\sqrt3-13$

$\frac{1}{2-\sqrt3}\times\frac{(2+\sqrt3)}{(2+\sqrt3)}=\frac{2+\sqrt3}{2-3}=-2-\sqrt3$

$\Longrightarrow 13\sqrt3-13-2-\sqrt3=$ جواب $\boxed{12\sqrt3-15}$

---

(۴)

الف) $\frac{P}{Q}=\frac{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}{\sqrt{\sqrt3-\sqrt2}}$, $P^2=1+\sqrt3+\sqrt3-1+2\sqrt{\frac{(1+\sqrt3)(\sqrt3-1)}{3-1=2}}$

$\frac{P^2}{Q}=\frac{2\sqrt3+2\sqrt2}{\sqrt{\sqrt3-\sqrt2}} \Rightarrow \frac{P}{Q}=\frac{\sqrt{2(\sqrt3+\sqrt2)}}{\sqrt{\sqrt3-\sqrt2}}=\sqrt{\frac{2(\sqrt3+\sqrt2)\times(\sqrt3+\sqrt2)}{\sqrt3-\sqrt2\times(\sqrt3+\sqrt2)}}=\sqrt{\frac{2(3+2+2\sqrt6)}{1}}=\sqrt{10+4\sqrt6}$

ب) $\sqrt[4]{\sqrt[3]{2^8}}\times\sqrt[4]{162}\times\sqrt{\frac{4}{2^2}\sqrt[6]{2}} \Rightarrow \sqrt[3]{2^{\frac{8}{4}}}=\sqrt[3]{2^2}\times3\sqrt[4]{2}\times\sqrt[12]{2^{13}}$

$2^{\frac{2\times4}{3\times4}+\frac{1\times3}{4\times3}+\frac{13}{12}}\times3 \Longrightarrow 2^{\frac{8+3+13}{12}}\times3=2^2\times3=12$

(5)

$$\frac{3^{x}+3^{x+1}+3^{x+2}+3^{x+3}+3^{x+4}+3^{x+5}}{2^{x-2}+2^{x-1}+2^{x}+2^{x+1}+2^{x+2}+2^{x+3}}=52 \qquad x=?$$

$$\Rightarrow a\left(\frac{q^{n}-1}{q-1}\right) \Rightarrow 1\left(\frac{3^{6}-1}{3-1}\right)=\frac{3^{6}-1}{2}=364$$

$$\frac{3^{x}(1+3^{1}+3^{2}+3^{3}+3^{4}+3^{5})}{2^{x}(2^{-2}+2^{-1}+1+2^{1}+2^{2}+2^{3})}=\frac{3^{x}(364)}{2^{x}\left(\frac{63}{4}\right)}=52 \Rightarrow \frac{52\times 63}{364\times 4}=\left(\frac{3}{2}\right)^{x}$$

$$\Rightarrow \frac{1}{4}\left(\frac{2^{6}-1}{2-1}\right)=\frac{2^{6}-1}{4}=\frac{63}{4}$$

$$\left(\frac{\cancel{52}^{\,1}\times\cancel{63}}{\cancel{364}\times 4}\right)=\left(\frac{3}{2}\right)^{x} \Rightarrow \left(\frac{9}{4}\right)=\left(\frac{3}{2}\right)^{x} \Rightarrow \left(\frac{3}{2}\right)^{2}=\left(\frac{3}{2}\right)^{x} \Rightarrow \boxed{x=2}$$

---

(6)

$$a=\sqrt[4]{\sqrt{6}-2}\ ,\quad b=\sqrt[4]{\sqrt{6}+2} \qquad t=?$$

$$(a^{2}+b^{2}-2ab)^{2}(a^{2}+b^{2}+2ab)^{2}=t(2-\sqrt{3})$$

$$\left((a-b)(a+b)\right)^{4}=t(2-\sqrt{3}) \Rightarrow (a^{2}-b^{2})^{4}=t(2-\sqrt{3})$$

$$\left(\sqrt[4]{(\sqrt{6}-2)^{2}}-\sqrt[4]{(\sqrt{6}+2)^{2}}\right)^{4} \Rightarrow \left(\left(\sqrt{\sqrt{6}-2}-\sqrt{\sqrt{6}+2}\right)^{2}\right)^{2} \Rightarrow \sqrt{6}\cancel{-2}+\sqrt{6}\cancel{+2}-2\sqrt{(\sqrt{6}-2)(\sqrt{6}+2)}$$

$$(2\sqrt{6}-2\sqrt{2})^{2}=t(2-\sqrt{3})=24+8-16\sqrt{3}=32-16\sqrt{3}=t(2-\sqrt{3})$$

$$16(\cancel{2-\sqrt{3}})=t(\cancel{2-\sqrt{3}}) \Rightarrow \boxed{t=6}$$

---

(7)

$$a=\sqrt[4]{7-4\sqrt{3}}\ ,\quad \left(a+\frac{1}{a}+\sqrt{2}\right)^{2}\left(a+\frac{1}{a}-\sqrt{2}\right)^{2}=2^{t} \qquad t=?$$

$$(2-\sqrt{3})^{2}=\sqrt{2-\sqrt{3}}$$

$$\left(\left(a+\frac{1}{a}+\sqrt{2}\right)\left(a+\frac{1}{a}-\sqrt{2}\right)\right)^{2}=2^{t} \Rightarrow \left(\left(a+\frac{1}{a}\right)^{2}-2^{1}\right)^{2}=2^{t} \Rightarrow$$

$$\left(\underbrace{2-\sqrt{3}+\frac{1}{2-\sqrt{3}}}\right)^{2}-2^{1}=2^{t} \Rightarrow (12-4)^{2}=2^{t} \Rightarrow 8^{2}=2^{t}$$

$$2-\sqrt{3}+\frac{2+\sqrt{3}}{\cancel{4-3}_{\,-1}}=\cancel{2}-\sqrt{3}-\cancel{2}-\sqrt{3}=(-2\sqrt{3})$$

$$2^{6}=2^{t} \longrightarrow \boxed{t=6}$$

جواب

۸

$A = \sqrt[۵]{\underbrace{\frac{۴}{۲^۲}\sqrt[۳]{\frac{۱۶}{۲^۴}}}_{\sqrt[۳]{\sqrt[۱۲]{۲^{+۰}}}\,۲}}\;\;\frac{(\frac{۱}{۲})^{-\frac{۴}{۳}}}{۲^{\frac{۴}{۳}}}$ $\qquad (۲۸)^{-\frac{۱}{۳}} = ? \rightarrow (\frac{۱}{۲})$

$\Longrightarrow A = ۲^{\frac{۲}{۳}} \times ۲^{\frac{۴}{۳}} \Rightarrow ۲^{\frac{۲}{۳}+\frac{۴}{۳}} = \frac{۶}{۳} \Rightarrow ۲^۲$

$(۲^۳)^{-\frac{۱}{۳}} = ۲^{-۱} \Rightarrow \frac{۱}{۲}$

۹

$\sqrt[۷]{a} = ۲۷\sqrt[۷]{a^{۱۵}}$ $\qquad \frac{\frac{۱}{a}-۳}{۱+\sqrt{۳}} = ?$

$a^{\frac{۱}{۷}} = ۳^۳ \times a^{\frac{۱۵}{۷}} \Rightarrow a^{\frac{۱}{۷}-\frac{۱۵}{۷}} = ۳^۳ \Rightarrow a^{-\frac{۱۴}{۷}} = ۳^۳ \Rightarrow (\frac{۱}{a})^۲ = ۲۷$

$a^{-۱} = ۳\sqrt{۳} \Rightarrow \frac{۱}{a} = ۳\sqrt{۳} \Longrightarrow \frac{۳\sqrt{۳}-۳}{۱+\sqrt{۳}} = \frac{۳(\sqrt{۳}-۱)\times(۱-\sqrt{۳})}{(۱+\sqrt{۳})\times(۱-\sqrt{۳})} \Rightarrow$

$\frac{۳(-۴+۲\sqrt{۳})}{۱-۳} \Rightarrow \frac{-۱۲+۶\sqrt{۳}}{-۲} = \frac{-\cancel{۶}^{۳}(۲-\sqrt{۳})}{-\cancel{۲}} = \boxed{۹-۳\sqrt{۳}}$ جواب

۱۰

$\underbrace{\sqrt{x+a}}_{a} - \underbrace{\sqrt{x-۴}}_{b} = ۲$ و $\underbrace{\sqrt{x+a}}_{a} + \underbrace{\sqrt{x-۴}}_{b} - ۲ = ? \rightarrow \frac{a}{۲}$

$(a-b) = ۲$ $\qquad \underbrace{(a+b)}_{\frac{a+۴}{۲}} - ۲ = \frac{a+۴}{۲} - \frac{۴}{۲} = \frac{a}{۲}$

$\underbrace{(a-b)}_{۲}(a+b) = a^۲-b^۲ \Rightarrow ۲(a+b) = a^۲-b^۲ \Rightarrow ۲(a+b) = \cancel{x}+a - \cancel{x}+۴$

$۲(a+b) = a+۴ \Rightarrow \boxed{a+b = \frac{a+۴}{۲}}$