هلیا عرب

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سوال ۱) الف) $(1!+3!+5!+\dots+25!)(2!+4!+6!+\dots)$

از ۵! به بعد یکان ۰ $\quad 1+(3\times2)=7$

از ۴! به بعد یکان ۰ $\quad 2+24=26 \to 6$

یکان $\quad 6\times7=42 \to 2$

سوال ۲) $\sqrt[4]{(4+\sqrt7)^{-1}} \times \sqrt{1+\sqrt7} \Rightarrow \sqrt[4]{\frac{1}{(4+\sqrt7)}\times2(4+\sqrt7)} = \sqrt[4]{2}$

$\sqrt[4]{(1+\sqrt7)^2} = \sqrt[4]{8+2\sqrt7}$

ب) $\left(\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\right)\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)$

$(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5})^2 = 3-\sqrt5+3+\sqrt5-4$

$6-4=2$ جذر $\pm\sqrt2$

$\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\times\frac{(\sqrt{10}-2)}{(\sqrt{10}-2)} = \frac{2\sqrt5-2\sqrt2+5\sqrt2-2\sqrt5}{6} = \frac{3\sqrt2}{6} = \frac{\sqrt2}{2}$

$-\sqrt2$ چون جواب پرانتز منفی است.

$+\frac{\sqrt2}{2}\times-\sqrt2=-1$ جواب

سوال ۳) الف) $\frac{\sqrt8+\sqrt{27}}{5-\sqrt6}\times\frac{(5+\sqrt6)}{(5+\sqrt6)} - 2(\sqrt[4]{9}-1)^{-1}$

$\frac{10\sqrt2+15\sqrt3+4\sqrt3+9\sqrt2}{19} = \frac{19\sqrt2+19\sqrt3}{19} = \sqrt2+\sqrt3$

$\frac{-2}{\sqrt3-1}\times\frac{\sqrt3+1}{(\sqrt3+1)} = \frac{-2\sqrt3-2}{2} = -\sqrt3-1$

$\sqrt2+\sqrt3-\sqrt3-1 = \sqrt2-1$

ب) $\frac{\sqrt{27}-1}{4+\sqrt3}\times\frac{(4-\sqrt3)}{(4-\sqrt3)} + (2-\sqrt3)^{-1}$

$\frac{12\sqrt3-4-9+\sqrt3}{13} = \sqrt3-1 + \frac{1}{2-\sqrt3}\times\frac{2+\sqrt3}{(2+\sqrt3)}$

$\sqrt3-1+2+\sqrt3 \Rightarrow 2\sqrt3+1$

سوال ۴) $\frac{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}{\sqrt{\sqrt3-\sqrt2}} - 2$ به توان ۲

$\frac{(\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1})^2}{\sqrt{\sqrt3-\sqrt2}} = \frac{1+\sqrt3+\sqrt3-1+2\sqrt2}{\sqrt3-\sqrt2}\times\frac{(\sqrt3+\sqrt2)}{(\sqrt3+\sqrt2)}$

$\frac{2\sqrt3+2\sqrt2}{1}\times(\sqrt3+\sqrt2) \Rightarrow \sqrt{\frac{2(\sqrt3+\sqrt2)}{1}} = \sqrt2\cdot(\sqrt3+\sqrt2)$

$\sqrt6+2-2=\sqrt6$

ب) $\sqrt[4]{\sqrt[3]{2^8}}\times\sqrt[4]{162}\times\sqrt{4\sqrt[6]{2}}$

$\sqrt[12]{2^8}\times3\sqrt[4]{2}\times2\sqrt[12]{2} \Rightarrow 2^{\frac{8}{12}}\times3\times2^{\frac14}\times2^{\frac1{12}}\times2 = 2^{\frac{12}{12}}\times6=12$

سوال ۵) $\frac{3^n+3^{n+1}+3^{n+2}+3^{n+3}+3^{n+4}+3^{n+5}}{2^{n-2}+2^{n-1}+2^n+2^{n+1}+2^{n+2}+2^{n+3}} = 52 \Rightarrow \frac{3^n(1+3+9+27+81+243)}{2^{n-2}(1+2+4+8+16+32)} = 52$

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$3^n\times364 = 52\times2^{n-2}\times63$

$3^n = 2^{n-2}\times9 \quad 3^n\div3^2 = 2^{n-2}$

$3^{n-2} = 2^{n-2}$

$n-2=0$

$n=2$

سوال ۶)

$(a^2+b^2-2ab)^2(a^2+b^2+2ab)^2 = t(2-\sqrt{3})$

$((a^2+b^2)^2-(2ab)^2)^2$

$(\sqrt{\sqrt{6}-2}+\sqrt{\sqrt{6}+2})^2$

$\sqrt{6}-2+\sqrt{6}+2\sqrt{2}+2-4\sqrt{2} = (2\sqrt{6}-2\sqrt{2})^2 = t(2-\sqrt{3})$

$24+8-16\sqrt{3} = t(2-\sqrt{3})$

32 ; $16(2-\sqrt{3}) = t(2-\sqrt{3})$

$t=16$

سوال ۷

$(a+\frac{1}{a}+\sqrt{2})^2(a+\frac{1}{a}-\sqrt{2})^2 = 2^t$

$((a+\frac{1}{a})^2-(\sqrt{2})^2)^2$; (-2)

$\sqrt[4]{7-4\sqrt{3}} = \sqrt[4]{(2-\sqrt{3})^2}$

$2-\sqrt{3}+\frac{1\times(2+\sqrt{3})}{2-\sqrt{3}\times(2+\sqrt{3})}+2 = 2-\sqrt{3}+2+\sqrt{3}+2 = 6-2 = 4$

$4^2=16$

$2^t = 2^4$

$t=4$

سوال ۸)

$A=\sqrt[5]{4\sqrt[3]{16}\,(\frac{1}{2})^{-\frac{4}{3}}}$; $2^{+\frac{4}{3}}$

$\sqrt[15]{2^{10}} \rightarrow 2^{\frac{10}{15}} \times 2^{\frac{4}{3}} = 2^{\frac{6}{3}} = 2^2 \rightarrow 4$

$(2(4))^{-\frac{1}{3}} = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$

سوال ۹) ریشه

$\sqrt[7]{a} = 27\times a^{\frac{15}{7}} \rightarrow 1 = 27\cdot a^2$

$a^2 = \frac{1}{27}$

$a = \frac{1}{3\sqrt{3}}$; $\frac{1}{a} = 3\sqrt{3}$

$\frac{3\sqrt{3}-3\times(1-\sqrt{3})}{1+\sqrt{3}\times(1-\sqrt{3})} =$

$\frac{3\sqrt{3}-9-3+3\sqrt{3}}{-2} = -3\sqrt{3}+6$

سوال ۱۰)

$\underbrace{\sqrt{x+a}}_{A}-\underbrace{\sqrt{x-4}}_{B} = 2$

$A-B=2$; $(A=B+2)$

$A^2 = B^2+4+4B$

$x+a = x-4+4+4\sqrt{x-4}$

$B=\frac{a}{4}$; $a=4B$

$\underbrace{\sqrt{x+a}}_{A}+\underbrace{\sqrt{x-4}}_{B} = -2 = ?$

$B+2+B-2=?$

$2B = ?$

$2(\frac{a}{4}) = \frac{a}{2}$