

$$(1! + 2! + 3! + \dots + 25!) (1! + 2! + 3! + \dots + 24!) = ?$$

$\underbrace{1! + 2! + 3! + \dots + 25!}_{(1)} \quad \underbrace{1! + 2! + 3! + \dots + 24!}_{(2)}$

چون: $\frac{1}{25!} = 2 \rightarrow 4 \times 25 = 100$

الف) $\sqrt[4]{(1+5\sqrt{2})^{-1}} \times \sqrt[4]{(1+5\sqrt{2})^2} \rightarrow \sqrt[4]{(1+5\sqrt{2})^{-1} \times (1+5\sqrt{2})^2}$

ب) $\sqrt[4]{\frac{(1+5\sqrt{2})^2}{1+5\sqrt{2}}} \rightarrow \sqrt[4]{\frac{1+5\sqrt{2}+25\sqrt{2}+50}{1+5\sqrt{2}}} \rightarrow \sqrt[4]{\frac{2(1+5\sqrt{2})}{1+5\sqrt{2}}} \rightarrow \sqrt[4]{2}$

چون: $\frac{1}{\sqrt{1+2}} \rightarrow \frac{1}{\sqrt{1+2}} \times \frac{\sqrt{1+2}}{\sqrt{1+2}} = \frac{\sqrt{1+2}}{1+2} = \frac{\sqrt{1+2}}{3}$

الف) $\frac{2\sqrt{2}+2\sqrt{2}}{5-5\sqrt{2}} \times \frac{5+5\sqrt{2}}{5+5\sqrt{2}} \rightarrow \frac{4\sqrt{2}+10\sqrt{2}}{19} \rightarrow \frac{14\sqrt{2}}{19}$

ب) $\frac{\sqrt{2}-1}{2+5\sqrt{2}} + (2-5\sqrt{2})^{-1} \rightarrow \frac{2\sqrt{2}-1}{2+5\sqrt{2}} \times \frac{2-5\sqrt{2}}{2-5\sqrt{2}} = \frac{4-10\sqrt{2}-2+10\sqrt{2}}{4-25} = \frac{-2}{21}$

چون: $\frac{1}{2-5\sqrt{2}} = \frac{2+5\sqrt{2}}{4-25} = \frac{2+5\sqrt{2}}{-21}$

الف) $\frac{\sqrt{2}+5\sqrt{2}+\sqrt{2}-1}{\sqrt{2}-5\sqrt{2}} \times \frac{\sqrt{2}+5\sqrt{2}}{\sqrt{2}+5\sqrt{2}} \rightarrow \frac{(2+10\sqrt{2}+2-1)}{(2-25)} \times \frac{\sqrt{2}+5\sqrt{2}}{\sqrt{2}+5\sqrt{2}} = \frac{(2+10\sqrt{2}+1)}{(2-25)} \times \frac{\sqrt{2}+5\sqrt{2}}{\sqrt{2}+5\sqrt{2}} = \frac{(3+10\sqrt{2})}{(-23)} \times \frac{\sqrt{2}+5\sqrt{2}}{\sqrt{2}+5\sqrt{2}} = \frac{(3+10\sqrt{2})(\sqrt{2}+5\sqrt{2})}{(-23)}$

ب) $\sqrt[12]{2^8} \times \sqrt[12]{2^4} \times \sqrt[12]{2^4} \rightarrow \sqrt[12]{2^8 \times 2^4 \times 2^4} = \sqrt[12]{2^{16}} = 2^{\frac{16}{12}} = 2^{\frac{4}{3}}$

صورت $\rightarrow 3^n (1+3+3^2+3^3+3^4+3^5) \rightarrow 3^n \times 342$

مخرج $\rightarrow 3^n (3^{-2}+3^{-1}+1+3+3^2+3^3) \rightarrow 3^n \times \frac{45}{2}$

چون: $\frac{3^n \times 342}{3^n \times \frac{45}{2}} = \frac{342}{\frac{45}{2}} = \frac{342 \times 2}{45} = \frac{684}{45} = \frac{4}{3}$

چون: $\frac{1}{3^n} = \frac{4}{3} \rightarrow n=2$

$$(a^2 + b^2 - rab)^r (a^2 + b^2 + rab)^r = t(r - \sqrt{r})$$

$$\frac{(a^2 + b^2)^r - (rab)^r}{(a^2 + b^2)^r} \Rightarrow (a^2 + b^2)^r \rightarrow (\sqrt{a-r} + \sqrt{a+r})^r = \frac{\sqrt{a-r} + \sqrt{a+r} + 2\sqrt{a-r}\sqrt{a+r}}{\sqrt{a-r}\sqrt{a+r}}$$

$$ab = \sqrt{(a-r)(a+r)} = \left(\frac{r}{2}\right)$$

$$\rightarrow (r(\sqrt{a+r}) - \frac{r}{2})^r = \frac{r}{2}(\sqrt{a-r})^r \rightarrow \frac{r}{2}(4+r-\sqrt{r}) = \frac{r}{2}(1-\sqrt{r}) \rightarrow r^2 - 14\sqrt{r} = t(r - \sqrt{r})$$

$$\rightarrow \frac{r}{2} \rightarrow \boxed{t=14}$$

$$\left(a + \frac{1}{a}\right)^r - (\sqrt{r})^r \rightarrow \left(a^r + \frac{1}{a^r} + r - r\right)^r = \left(a^r + \frac{1}{a^r}\right)^r$$

$$a \Rightarrow \sqrt{r-\sqrt{r}} = \sqrt{r-\sqrt{r}}$$

$$(\sqrt{r-\sqrt{r}})^r + \frac{1}{(\sqrt{r-\sqrt{r}})^r} = \left(r - \sqrt{r} + \frac{1}{r - \sqrt{r}}\right)^r = \left(r - \sqrt{r} + r + \sqrt{r}\right)^r = (2r)^r = 14 \rightarrow r^2$$

$$\rightarrow \frac{r}{2} \rightarrow \boxed{t=4}$$

$$\sqrt[r]{x^r \cdot r^r} \rightarrow \sqrt[r]{x^r} \rightarrow \sqrt[r]{r^r} \rightarrow \left(\frac{1}{r}\right)^{-\frac{r}{r}} \rightarrow \sqrt[r]{\left(\frac{1}{r}\right)^{-r}} \rightarrow \sqrt[r]{\frac{1}{r^r}}$$

$$A = \sqrt[r]{x^r \cdot r^r} \rightarrow \sqrt[r]{r^r} \rightarrow \boxed{r^r = A} \rightarrow (rA)^{-\frac{1}{r}} \rightarrow (1)^{-\frac{1}{r}} \rightarrow \left(\frac{1}{r}\right)$$

$$a^{\frac{1}{r}} = r \cdot v \quad a^{\frac{1}{r}} \rightarrow a^{-r} = r \cdot v \rightarrow a^{-1} = \sqrt{r \cdot v} \rightarrow \frac{1}{a} = \sqrt{r \cdot v} = r \cdot \sqrt{r}$$

$$\frac{r\sqrt{r}-r}{\sqrt{r}+1} = \frac{r(\sqrt{r}-1)}{\sqrt{r}+1} \times \frac{\sqrt{r}-1}{\sqrt{r}-1} = \frac{r}{r} \left(\frac{r+1-r\sqrt{r}}{9} \right)$$

$$\rightarrow \frac{r}{r} \times r (r - \sqrt{r})$$

$$\frac{\sqrt{n+a} - \sqrt{n-r}}{m} = r \quad / \quad \frac{\sqrt{n+a} + \sqrt{n-r}}{n} = r \quad ? \rightarrow \frac{N+r}{m} + \frac{N-r}{n} = ?$$

$$m - n = r \rightarrow \boxed{m = n + r} \rightarrow \frac{1}{r} \times \frac{r}{r} = \frac{1}{r} \times \frac{r}{r} \rightarrow a = \frac{a}{r} \rightarrow \boxed{N = \frac{a}{r}}$$

$$r \times \frac{a}{r} = \left(\frac{a}{r}\right)$$