

۱۴, ۲۵

$$\underbrace{(1 + \sqrt{2} + \sqrt{3} + \dots)}_{\sqrt{2} = ۱۴} \times \underbrace{(1 + \sqrt{2} + \sqrt{3} + \dots)}_{\sqrt{2} = ۱۴} = \dots$$

۲ کان

~~$$\left( \frac{\sqrt{2} + \sqrt{3}}{\sqrt{10} + 1} \right) \times \frac{\sqrt{10} - 1}{\sqrt{10} - 1} = \frac{3\sqrt{2}}{4} \times \frac{\sqrt{2}}{2}$$~~
~~$$\left( \sqrt{2} - \sqrt{3} - \sqrt{3} + \sqrt{5} \right) \times \frac{\sqrt{2}}{2} = \frac{\sqrt{4 - 2\sqrt{5}} - \sqrt{4 + 2\sqrt{5}}}{2}$$~~

الف)  $\sqrt{(1 + \sqrt{2})^2} = \sqrt{1 + 2 + 2\sqrt{2}} \times \sqrt{(1 + \sqrt{2})^{-1}}$

$$(1 + \sqrt{2}) \times \frac{1}{1 + \sqrt{2}} \times \frac{1 - \sqrt{2}}{1 - \sqrt{2}} = 1$$

ب)  $\sqrt{2}$

~~$$\frac{3\sqrt{3} - 1}{1 + \sqrt{3}} \times \frac{1 - \sqrt{3}}{1 - \sqrt{3}} = \frac{13\sqrt{3} - 13}{13} = \sqrt{3}$$~~
~~$$\frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} = \frac{2 + \sqrt{3}}{4 - 3}$$~~
~~$$2 + \sqrt{3} + \sqrt{3} - 1 = 1 + 2\sqrt{3}$$~~

ب)  $\frac{\sqrt{1 + \sqrt{2}}}{5 - \sqrt{4}} \times \frac{5 + \sqrt{4}}{5 + \sqrt{4}} = \frac{(\sqrt{1 + \sqrt{2}})(5 + \sqrt{4})}{19}$

$$\frac{10\sqrt{2} + 15\sqrt{3} + 5\sqrt{3} + 9\sqrt{2}}{19} = \sqrt{2} + \sqrt{3}$$

$$\frac{\sqrt{2} + \sqrt{3} - 2}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} = \frac{\sqrt{2} + \sqrt{3} - 13 - 1}{4} = \frac{\sqrt{2} - 1}{4}$$

~~$$\sqrt[12]{2^8} \times \sqrt[12]{2^4} \times \sqrt[12]{2^8} = \sqrt[12]{2^{20}}$$~~
~~$$\sqrt[12]{2^8 \times 2^4 \times 2^8} = \sqrt[12]{2^{20}} = 2 \times 2 = 4$$~~

الف)  $\frac{\sqrt{1 + \sqrt{2}} + \sqrt{2} - 1}{\sqrt{2} - \sqrt{2}} \times \frac{\sqrt{2} + \sqrt{2}}{\sqrt{2} + \sqrt{2}} =$

~~$$\frac{\sqrt{4 + 2\sqrt{2}} - 2}{2}$$~~

$$\frac{3^x(1 + 3 + 9 + 27 + 81)}{3^{x-2}(1 + 3 + 9 + 27 + 81)} = 3^2 \rightarrow 3^x \times 3^4 = 3^{x-2} \times 3^4$$

$$3^x \times 3^4 = 3^{x-2} \times 3^4 \rightarrow 3^x = 3^{x-2} \rightarrow x = x - 2 \rightarrow x = 2$$

$$3^x \times 3^4 = 3^x \times 81 \rightarrow 3^x \times 91 = 3^x \times 91 \rightarrow 3^x = 3^x$$

$$x - 2 = 0 \rightarrow x = 2$$

$$(a-b)^{\frac{1}{2}}(a+b)^{\frac{1}{2}} = (a^{\frac{1}{2}} - b^{\frac{1}{2}})^{\frac{1}{2}} = (2\sqrt{4} - 2\sqrt{2})^{\frac{1}{2}} = 2^{\frac{1}{2}} + 1 - \sqrt{12}$$

$$22 - 14\sqrt{3} = 14(2 - \sqrt{3}) \rightarrow t = 14$$

$$\left[ \left( a + \frac{1}{a} + \sqrt{2} \right) \left( a + \frac{1}{a} - \sqrt{2} \right) \right]^{\frac{1}{2}} = \left[ a^{\frac{1}{2}} + \frac{1}{a^{\frac{1}{2}}} + 1 - 1 \right]^{\frac{1}{2}} = a^{\frac{1}{2}} + \frac{1}{a^{\frac{1}{2}}} + 1$$

$$\sqrt{2 - 2\sqrt{3}} + \frac{1}{\sqrt{2 - 2\sqrt{3}}} + 1 = \sqrt{2 - 2\sqrt{3}} + \sqrt{2 + 2\sqrt{3}} + 1 = 14 \Rightarrow t = 14$$

$$\sqrt[10]{2^{\frac{10}{3}} \times 2^{\frac{1}{3}}} \times (2)^{\frac{1}{3}} \rightarrow \sqrt[10]{2^{\frac{10}{3}}} \times \sqrt[3]{2^{\frac{1}{3}}} = \sqrt[10]{2^{\frac{10}{3}} \times 2^{\frac{10}{3}}} = 2$$

$$(2 \times 2)^{-\frac{1}{2}} = \left( \frac{1}{2} \right)^{\frac{1}{2}} \Rightarrow 2^{-1} = \left( \frac{1}{2} \right)$$

$$\sqrt[3]{a} = 2\sqrt[3]{a} \rightarrow a = 2^{\frac{3}{2}} a^{\frac{1}{2}} \rightarrow \frac{1}{a} = \frac{1}{2^{\frac{3}{2}} a^{\frac{1}{2}}}$$

$$1 = 2^{\frac{3}{2}} a^{\frac{1}{2}} \rightarrow a^{\frac{1}{2}} = \frac{1}{2^{\frac{3}{2}}} \rightarrow a = \frac{1}{2^3} \rightarrow \frac{1}{a^2} = 2^6 \rightarrow \frac{1}{a} = \sqrt{2^6} = 2^3 = 8$$

$$\frac{2\sqrt{3} - 2}{1 + \sqrt{3}} \times \frac{1 - \sqrt{3}}{1 - \sqrt{3}} = \frac{2\sqrt{3} - 2 - 2 + 2\sqrt{3}}{-2} = 3 - 3\sqrt{3}$$

$$\sqrt{x+a} - 2 = \sqrt{x-2} \rightarrow \sqrt{x+a} + \sqrt{x+a} - 2 - 2 = \sqrt{x-2} + 1 + \sqrt{x-2} + 1 - 2$$

$$2\sqrt{x+a} - 2 = 2\sqrt{x-2} \rightarrow \sqrt{x+a} - 1 = \sqrt{x-2} \rightarrow 2\sqrt{x-2} = 2\sqrt{x+a-1}$$