

$$1) (1! + 3! + 5! + \dots + 25!) (2! + 4! + 6! + \dots + 26!)$$

$$1! + 3! = 7$$

$$2! + 4! = 26 \rightarrow \text{چون}$$

$$\Rightarrow 7 \times 26 = 182 \rightarrow \text{چون}$$

جواب نهایی

$$2) \text{الف)} \sqrt[4]{(4+\sqrt{7})^{-1}} \times \sqrt{1+\sqrt{7}} = \sqrt[4]{(4+\sqrt{7})^{-1}} \times \sqrt{(1+\sqrt{7})^2} = \sqrt[4]{(4+\sqrt{7})^{-1}} \times \sqrt{8+2\sqrt{7}} = \sqrt[4]{\frac{8+2\sqrt{7}}{4+\sqrt{7}}} = \sqrt[4]{2}$$

$$\text{ب)} \left(\frac{\sqrt{2}+\sqrt{5}}{\sqrt{10}+2} \right) (\sqrt{3}-\sqrt{5}-\sqrt{3}+\sqrt{5}) = \left(\frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}(\sqrt{5}+\sqrt{2})} \right) (\sqrt{3}-\sqrt{5}-\sqrt{3}+\sqrt{5}) =$$

$$\frac{\sqrt{3}-\sqrt{5}-\sqrt{3}+\sqrt{5}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}-\sqrt{10}-\sqrt{6}+\sqrt{10}}{2} = \frac{\sqrt{(\sqrt{5}-1)^2} - \sqrt{(\sqrt{5}+1)^2}}{2} = \frac{\sqrt{5}-1-\sqrt{5}-1}{2} = -1$$

$$3) \text{الف)} \frac{\sqrt{8}+\sqrt{27}}{5-\sqrt{6}} - 2(\sqrt[4]{9}-1)^{-1} = \frac{2\sqrt{2}+3\sqrt{3}}{(\sqrt{2})^2+(\sqrt{3})^2-\sqrt{3}\times\sqrt{2}} - \frac{2}{\sqrt{3}-1}$$

$$\frac{(2\sqrt{2}+3\sqrt{3})(\sqrt{2}+\sqrt{3})}{2\sqrt{2}+3\sqrt{3}} - \frac{2(\sqrt{3}+1)}{\sqrt{3}-1} = \sqrt{2}+\sqrt{3}-\sqrt{3}-1 = \sqrt{2}-1$$

$$\text{ب)} \frac{\sqrt{27}-1}{4+\sqrt{3}} + (2-\sqrt{3})^{-1} = \frac{3\sqrt{3}-1}{1+3-\sqrt{3}} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} + \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{(3\sqrt{3}-1)(\sqrt{3}-1)}{3\sqrt{3}-1} + 2+\sqrt{3}$$

$$\sqrt{3}-1+2+\sqrt{3} = 2\sqrt{3}+1$$

(الف)

$$f) \frac{\sqrt{1+\sqrt{3}} + \sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}} \cdot \sqrt{1+\sqrt{3}} + \sqrt{\sqrt{3}-1} = \frac{\sqrt{1+\sqrt{3}} + \sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}} \times \frac{\sqrt{\sqrt{3}+\sqrt{2}}}{\sqrt{\sqrt{3}+\sqrt{2}}} = (\sqrt{1+\sqrt{3}} + \sqrt{\sqrt{3}-1})(\sqrt{\sqrt{3}+\sqrt{2}}) =$$

$$\sqrt{[(\sqrt{1+\sqrt{3}} + \sqrt{\sqrt{3}-1})(\sqrt{\sqrt{3}+\sqrt{2}})]^2} = \sqrt{(1+\sqrt{3}-\sqrt{2})(\sqrt{3}-\sqrt{2})} = (\sqrt{3}-\sqrt{2})\sqrt{2} = \sqrt{2}(\sqrt{3}-\sqrt{2}) = \sqrt{2}\sqrt{3}-\sqrt{2}\sqrt{2} = \sqrt{6}-2$$

$$b) \sqrt[5]{x^5 \sqrt{x}} \times \sqrt[5]{16x} \times \sqrt[5]{x^5 \sqrt{x}} = x^{\frac{5}{5}} \times x^{\frac{1}{5}} \times x^{\frac{5}{5}} \times x^{\frac{1}{5}} = x \times x = x^2 = \boxed{16}$$

$$d) \frac{x^2 + x^{2+1} + x^{2+2} + x^{2+3} + x^{2+4}}{x^{2-1} + x^{2-2} + x^2 + x^{2+1} + x^{2+2} + x^{2+3}} = \Delta x \rightarrow \frac{x^2(1+x+9+\dots+x^5)}{x^{2-1}(1+x+x+\dots+x^5)} =$$

$$\frac{x^2(\frac{x^6-1}{x-1})}{x^{2-1}(\frac{x^6-1}{x-1})} = \frac{x^2(x^5-1)}{x \times x^{2-1}(x^5-1)} = \frac{x^2 \times x^5}{x^3 \times x^5} = \Delta x \rightarrow 13 \times x^2 \rightarrow \frac{x^{2-1}}{x^{2-1}} = 1 \rightarrow x = \boxed{13}$$

$$g) (a^r + b^r - r a^r b^r)^r (a^r + b^r + r a^r b^r)^r = ((a^r + b^r)^r - r a^r b^r)^r ((a^r + b^r)^r + r a^r b^r)^r = (a^r + b^r - r a^r b^r)^r = \left[(\sqrt{\sqrt{5}-2})^r + (\sqrt{\sqrt{5}+2})^r - r(\sqrt{\sqrt{5}-2})(\sqrt{\sqrt{5}+2}) \right]^r =$$

$$(r\sqrt{5}-r\sqrt{2})^r = [r\sqrt{r}(\sqrt{5}-1)]^r = 16(r-\sqrt{5}) = t(r-\sqrt{5}) \rightarrow \underline{t=16}$$

$$v) a - \sqrt[5]{x^5 \sqrt{x}} = \sqrt{(x-\sqrt{x})^2} = \sqrt{x-\sqrt{x}} \rightarrow (a + \frac{1}{a} + \sqrt{x})^r (a + \frac{1}{a} - \sqrt{x})^r =$$

$$((a + \frac{1}{a})^r - r)^r = (a^r + \frac{1}{a^r} + r - r)^r = (a^r + \frac{1}{a^r})^r = (x - \sqrt{x} + \frac{1}{x-\sqrt{x}})^r =$$

$$(x - \sqrt{x} + x + \sqrt{x})^r = 16 = x^t \Rightarrow \underline{t=5}$$

$$h) A = \sqrt[5]{x^5 \sqrt{16}} \times (\frac{1}{x})^{-\frac{5}{5}} = x^{\frac{5}{5}} \times x^{\frac{5}{5}} = x \rightarrow (xA)^{-\frac{1}{5}} = A^{-\frac{1}{5}} = \frac{1}{x}$$

$$9) x \sqrt[5]{a^{\frac{10}{5}}} = a^{\frac{1}{5}} \rightarrow x \sqrt[5]{x a^r} = 1 \rightarrow a = \frac{1}{x \sqrt[5]{x}}$$

$$\hookrightarrow \frac{\frac{1}{a} - x}{1 + \sqrt{x}} = \frac{x\sqrt{x} - x}{1 + \sqrt{x}} \times \frac{\sqrt{x}-1}{\sqrt{x}-1} = \frac{x(x-\sqrt{x})}{x} = \underline{\underline{x-\sqrt{x}}}$$

((ب))