۲۰

نام و نام خانوادگی: نگار مقبل   شماره کلاس: ۲۱

۱) $(1!+3!+5!+\dots+25!)(2!+4!+6!+\dots+26!)$

$(1+6+0+0+\dots+0)(2+4+0+0+\dots+0)=42$

$\quad\;\, 7 \qquad\qquad\qquad\quad 6$

رقم یکان $=2$

۲) الف) $\sqrt[4]{(4+\sqrt7)^{-1}}\cdot\sqrt[4]{(1+\sqrt7)^2}=\sqrt[4]{\dfrac{8+2\sqrt7}{4+\sqrt7}}=\sqrt[4]{2}$

ب) $\left(\dfrac{\sqrt2+\sqrt{10}}{2}\right)\left(\dfrac{\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}}{\sqrt2}\right)\times\dfrac{\sqrt2}{\sqrt2}=$

$\dfrac{\sqrt{6-2\sqrt5}-\sqrt{6+2\sqrt5}}{2}=\dfrac{\sqrt5-1-\sqrt5-1}{2}=-1$

۳) الف) $\dfrac{2\sqrt2+3\sqrt3}{5-\sqrt6}-\dfrac{2}{\sqrt3-1}=\sqrt3+\sqrt2-\sqrt3-1=\sqrt2-1$

$\dfrac{10\sqrt2+4\sqrt3+15\sqrt3+9\sqrt2}{19}=\sqrt3+\sqrt2$ ، $\dfrac{2\sqrt3+2}{2}=\sqrt3+1$

ب) $\dfrac{3\sqrt3-1}{4+\sqrt3}+\dfrac{1}{2-\sqrt3}$

$\dfrac{12\sqrt3-9-4+\sqrt3}{13}+\dfrac{2+\sqrt3}{1}=\sqrt3-1+2+\sqrt3=2\sqrt3+1$

۴) الف) $\dfrac{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}{\sqrt{\sqrt3-\sqrt2}}\times\dfrac{\sqrt{\sqrt3+\sqrt2}}{\sqrt{\sqrt3+\sqrt2}}=\left(\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}\right)\sqrt{\sqrt3+\sqrt2}$

$\sqrt{\left(\left(\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}\right)\left(\sqrt{\sqrt3+\sqrt2}\right)\right)^2}=\sqrt{(2\sqrt3+2\sqrt2)(\sqrt3-\sqrt2)}=\sqrt{(\sqrt6-2)^2}$

$\sqrt6-2-2=\sqrt6-4$

۱۱) ب) $\sqrt[3\times4]{4^{4}} \times 3\sqrt[4\times3]{2^{3}} \times 2\sqrt[3]{2} = \sqrt[3]{2^{8}} \times 3\sqrt[3]{2^{3}} \times 2\sqrt[3]{2} = 9\sqrt[3]{2^{4}}$

۱۵)

$$\frac{3^x(1+3+9+27+81+243)}{2^x(\frac{1}{4}+\frac{2}{4}+\frac{4}{4}+\frac{8}{4}+\frac{16}{4}+\frac{32}{4})} = 567$$

$364 \quad 28 \quad \frac{63}{4}$

$$\left(\frac{3}{2}\right)^x = \frac{63}{28} = \frac{9}{4} \qquad x=2$$

۱۶)

$$(a^2+b^2-2ab)^2(a^2+b^2+2ab)^2 = ((a^2+b^2)^2-4a^2b^2)^2$$

$$(a^4+b^4+2a^2b^2-4a^2b^2)^2 = (a^4+b^4-2a^2b^2)^2 =$$

$$\left[(\sqrt[4]{\sqrt{6}-2})^4 + (\sqrt[4]{\sqrt{6}+2})^4 - 2(\sqrt{\sqrt{6}-2})(\sqrt{\sqrt{6}+2})\right]^2 =$$

$2\sqrt{6} \qquad \sqrt{2}$

$$(2\sqrt{6}-2\sqrt{2})^2 = \left[2\sqrt{2}(\sqrt{3}-1)\right]^2 = 16(2-\sqrt{3}) = t(2-\sqrt{3})$$

$$t=16$$

۱۷)

$$\left(a+\frac{1}{a}+\sqrt{2}\right)^2\left(a+\frac{1}{a}-\sqrt{2}\right)^2 = 2^t$$

$$a = \sqrt[4]{7-4\sqrt{3}} = \sqrt[4]{(2-\sqrt{3})^2} = \sqrt{2-\sqrt{3}}$$

$$\left(\sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}}+\sqrt{2}\right)^2\left(\sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}}-\sqrt{2}\right)^2 = 2^t$$

$$(2-\sqrt{3}+2+\sqrt{3}+2+2+2\sqrt{4+2\sqrt{3}}+2\sqrt{4-2\sqrt{3}})$$

$$(8+2\sqrt{4+2\sqrt{3}}+2\sqrt{4-2\sqrt{3}})(8-2\sqrt{4+2\sqrt{3}}-2\sqrt{4-2\sqrt{3}})$$

$$64-(16+8\sqrt{3}+16-8\sqrt{3}+16) = 16 = 2^t \qquad t=4$$

$A = 2^{\frac{2}{5}=\frac{4}{10}} \times 2^{\frac{4}{10}} \times 2^{\frac{4}{15}} = 2^{\frac{2}{15}} \times 2^{\frac{4}{15}} = 2^{2} = 4$ (8

$(2A)^{-\frac{1}{3}} = (2^{3})^{-\frac{1}{3}} = \frac{1}{2}$

$\sqrt[2]{a} = 27a^{\frac{5}{2}}$ (9

$\frac{a^{\frac{1}{2}}}{a^{\frac{5}{2}}} = 3^{3}$ $\quad a^{-2} = 3^{3}$ $\quad a > 0$

$(\frac{1}{a})^{2} = 3^{3}$ $\quad a^{2} = \frac{1}{27} \Rightarrow a = \frac{1}{3\sqrt{3}}$

$3(\sqrt{3}-1)$ $\quad 12-6\sqrt{3}$

$\frac{3\sqrt{3}-3}{1+\sqrt{3}} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{3(3+1-2\sqrt{3})}{2} = 6-3\sqrt{3}$

$\underbrace{(\sqrt{x+a}-\sqrt{x-4})}_{2}(\sqrt{x+a}+\sqrt{x-4}) = x+a-x+4$ (10

$= a+4$

$\sqrt{x+a}+\sqrt{x-4} = \frac{a}{2}+2$

$\sqrt{x+a}+\sqrt{x-4}-2 = \frac{a}{2}+2-2 = \boxed{\frac{a}{2}}$