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نام و نام خانوادگی ....[illegible]........ پاسخنامه تشریحی تکلیف شماره ..۲۱.... کلاس .....دهم تجربی ۸.........

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رقم یکان : ۲

$(1!+3!+5!+\dots+25!)(2!+4!+6!+\dots+24!) =$

$1!+3!+5!+\dots+25!$: 1 ، 6 ، یکان $=0$ ، 0 → یکان $=7$

$2!+4!+\dots+24!$: $2+24+\dots = 26$ ، یکان $=0$ → یکان $=6$

$6\times7=42$

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الف) $\sqrt[4]{(4+\sqrt7)^{-1}}\times\sqrt{1+\sqrt7} = \sqrt[4]{(4+\sqrt7)^{-1}\times(1+\sqrt7)^2}$ ، $(1+\sqrt7)^2=(8+2\sqrt7)=2(4+\sqrt7)$

$=\sqrt[4]{(4+\sqrt7)^{-1}\,2\,(4+\sqrt7)^{1}} = \boxed{\sqrt[4]{2}}$

ب) $\left(\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\times\frac{\sqrt{10}-2}{\sqrt{10}-2}\right)\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)$

$\frac{(\sqrt2+\sqrt5)(\sqrt{10}-2)}{10-4} = \frac{\cancel{2\sqrt5}-2\sqrt2+5\sqrt2-\cancel{2\sqrt5}}{6} = \frac{\sqrt2}{2}\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)$

$=\frac{\sqrt{6-2\sqrt5}-\sqrt{6+2\sqrt5}}{2} = \frac{\sqrt{(\sqrt5-1)^2}-\sqrt{(\sqrt5+1)^2}}{2} = \frac{-2}{2} = \boxed{-1}$

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الف) $\frac{\sqrt8+\sqrt{27}}{5-\sqrt6} - 2\left(\sqrt[4]{9}-1\right)^{-1} = \sqrt2+\sqrt3-(\sqrt3+1) = \boxed{\sqrt2-1}$

$\frac{2\sqrt2+3\sqrt3}{5-\sqrt6}\times\frac{5+\sqrt6}{5+\sqrt6} = \frac{10\sqrt2+\cancel{2\sqrt{12}}+15\sqrt3+\cancel{3\sqrt{18}}}{25-6} = \frac{19\sqrt2+19\sqrt3}{19} = \sqrt2+\sqrt3$

$\sqrt[4]{3^2} = (\sqrt3-1)^{-1}\times2 = \frac{2}{\sqrt3-1} = \frac{2\sqrt3+2}{2} = \boxed{\sqrt3+1}$

ب) $\frac{\sqrt{27}-1}{4+\sqrt3}+(2-\sqrt3)^{-1} = \frac{3\sqrt3-1}{4+\sqrt3}\times\frac{(4-\sqrt3)}{(4-\sqrt3)} = \frac{12\sqrt3-4-9+\sqrt3}{16-3} = \frac{13(\sqrt3-1)}{13} \to \sqrt3-1+\sqrt3+2 =$

$(2-\sqrt3)^{-1} = \frac{1}{2-\sqrt3} = 2+\sqrt3$

$\boxed{2\sqrt3-1}$

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الف) $\frac{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}{\sqrt{\sqrt3-\sqrt2}}-2=A$

$A^2=\frac{\cancel{1}+\sqrt3+\sqrt3-\cancel{1}+2\sqrt2}{\sqrt3-\sqrt2} = \frac{2\sqrt3+2\sqrt2}{\sqrt3-\sqrt2}\times\frac{\sqrt3+\sqrt2}{\sqrt3+\sqrt2} = \frac{2(\sqrt3+\sqrt2)^2}{1} = 2(5+2\sqrt6) = (A+2)^2$

$2(\sqrt3+\sqrt2)^2=(A+2)^2 \to \sqrt2(\sqrt3+\sqrt2)=A+2$

$\sqrt6+\cancel{2}=A+\cancel{2}$

$\boxed{A=\sqrt6}$

ب) $\sqrt[4]{\sqrt[3]{2^8}}\times\sqrt[4]{192}\times\sqrt{4\sqrt[6]{2}} =$

$\sqrt[12]{2^8}\times 3\sqrt[4]{2}\times\sqrt[12]{2^{13}}$ ، $192 = 2^6\times3$

$=3\times\sqrt[12]{2^8\times2^3\times2^{13}} = 2^{24}$ $= 2^2\times3=12$

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$\frac{3^x+3^{x+1}+3^{x+2}+3^{x+3}+3^{x+4}+3^{x+5}}{2^{x-2}+2^{x-1}+2^x+2^{x+1}+2^{x+2}+2^{x+3}}=52$

$x=2$

$\cancel{819}\times2^x=\cancel{364}\times3^x$

$\frac{3^x(1+3+9+27+81+243)}{2^{x-2}(1+2+2^2+2^3+2^4+2^5)}=52 \quad = \frac{364\times3^x}{15{,}75\times2^x}=52 \quad \left(\frac32\right)^x=2{,}25$

$3+4+8+16+32$

$63\times2^x\times\frac14$

$\boxed{x=2}$

$a = \sqrt[4]{\sqrt{6}-2}$ $\qquad t = 16$ $\qquad$ 3

$b = \sqrt[4]{\sqrt{6}+2}$

$(a^2+b^2-2ab)^2(a^2+b^2+2ab)^2 = \overset{\to 16}{t}(2-\sqrt{3})$ $\qquad (2\sqrt{6}-2\sqrt{2})^2 = \cancel{24}_{32}+8-8\sqrt{12} = 8(4-\underset{2\sqrt{3}}{\sqrt{12}}) = 16(2-\sqrt{3})$ $\qquad$ 6

$\overset{2(2-\sqrt{3})}{}$

$((a-b)^2)^2((a+b)^2)^2$

$[(a-b)(a+b)]^4 = (a^2-b^2)^4 = (\sqrt{\sqrt{6}-2}-\sqrt{\sqrt{6}+2})^4 = ((\sqrt{\sqrt{6}-2}-\sqrt{\sqrt{6}+2})^2)^2 = \sqrt{6}\cancel{-2}+\sqrt{6}\cancel{+2}-2\sqrt{2}$

$t = 4$

$a = \sqrt[4]{7-4\sqrt{3}} \to (\sqrt{3}-2)^2 = \sqrt{\sqrt{3}-2}$

$(a+\frac{1}{a}+\sqrt{2})^2(a+\frac{1}{a}-\sqrt{2})^2 = 2^t$ $\qquad$ 2 $\qquad$ 7

$[(a+\frac{1}{a}+\sqrt{2})(a+\frac{1}{a}-\sqrt{2})]^2$

$((a^2+\frac{1}{a^2}+2)-2)^2 = (a^2+\frac{1}{a^2})^2 = (\sqrt{3}-2+\frac{1}{\sqrt{3}-2})^2 = \frac{\cancel{3+4+1}^{\,8}-4\sqrt{3}}{\sqrt{3}-2} = \frac{-4(\sqrt{3}-2)}{(\sqrt{3}-2)} = (-4)^2 = +16 = 2^4$

$A = \sqrt[5]{4\sqrt[3]{16}}\,(\frac{1}{2})^{-\frac{4}{3}} = \sqrt[15]{2^{10}} \times \sqrt[3]{2^4} = \sqrt[15]{2^{10}\times 2^{20}} = \sqrt[15]{2^{30}} = 2^2 = 4$

$(2A)^{-1/3} = (8)^{-1/3} = \sqrt[3]{8^{-1}} = \sqrt[3]{\frac{1}{8}} = \boxed{\frac{1}{2}}$ $\qquad$ 3 $\qquad$ 8

$\sqrt[7]{a} = 27a^{\frac{15}{7}} \to 27\sqrt[7]{\underset{a^{14}\times a}{a^{15}}} = \sqrt[7]{a} \to 27\times a^2\cancel{\sqrt[7]{a}} = \cancel{\sqrt[7]{a}}\ 1 \to 27a^2 = 1$

$a^2 = \frac{1}{27}$

$\overset{\to a^{-1}}{\frac{1}{a}}-3 = x(1+\sqrt{3})$ $\qquad 3-3\sqrt{3}-\sqrt{3}+3 = 6-2\sqrt{3}$ $\qquad 2(3-\sqrt{3})$

گویا $= \frac{3(1-\sqrt{3})(3-\sqrt{3})}{9-3} = \frac{\cancel{6}(3-\sqrt{3})}{\cancel{6}} = \boxed{3-\sqrt{3}}$ — 2 $\qquad a = \frac{1}{3\sqrt{3}} = \frac{3\sqrt{3}}{3\cancel{9}}\ 9\boxed{\frac{\sqrt{3}}{3}}$

$x = ?$ $\qquad \to 3(1-\sqrt{3})$

$\frac{3}{\sqrt{3}}-3 = \frac{3-3\sqrt{3}}{\sqrt{3}} = \frac{3(1-\sqrt{3})}{\sqrt{3}} = x\times(1+\sqrt{3}) \to x = \frac{3(1-\sqrt{3})}{\sqrt{3}(1+\sqrt{3})} = \frac{3(1-\sqrt{3})}{3+\sqrt{3}}\ \boxed{\frac{3-3\sqrt{3}}{3+\sqrt{3}}}$

$\underbrace{\sqrt{x+a}}_{v'} - \underbrace{\sqrt{x-4}}_{v} = 2$ $\qquad v'-v = 2$ $\qquad$ 3

$v'^2-v^2 = a+4 = (v'-v)(v'+v) \to v'+v = \frac{a+4}{2}$ $\Big\}\ 2v' = \frac{a+4+4}{2} = \frac{a+8}{4}$ $\qquad$ 10

$\underbrace{\sqrt{x+a}+\sqrt{x-4}}-2 =$

$\frac{a+4}{2}-2 = \frac{a}{2}$