آتریسا آقاصالحی ۲۰ ۲۱

۱) عامل ۵ ۲×۵ ← !۶ , !۵

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$1! = 1 \quad 3! = 6 \longrightarrow 1+6=7$

$2! = 2 \quad 4! = 24 \longrightarrow 2+24 = 26$

$6 \times 7 = 42$ ← یکان

۲) الف)

$$\frac{1}{\sqrt[4]{4+\sqrt{7}}} \times \frac{\sqrt[4]{(1+\sqrt{7})^2}}{1} \rightarrow 8+2\sqrt{7}$$

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$$= \frac{\sqrt[4]{2(4+\sqrt{7})}}{\sqrt[4]{4+\sqrt{7}}} = \boxed{\sqrt[4]{2}}$$

ب)

$$\frac{\sqrt{6-2\sqrt{5}}}{\sqrt{2}} = \frac{\sqrt{3}\times\sqrt{3-\sqrt{5}}}{\sqrt{2}} \quad \frac{\sqrt{2+\sqrt{5}}}{\sqrt{2}(\sqrt{5}+\sqrt{2})}$$

$\cancel{\sqrt[4]{(1-\sqrt{5})^2}}$

$$\frac{\sqrt{2}\times -\sqrt{3+\sqrt{5}}}{\sqrt{2}} = \frac{-(1+\sqrt{5})}{\sqrt{2}} \qquad = \frac{-2}{2} = -1$$

۳) الف)

$$\frac{2\sqrt{2}+3\sqrt{3}}{5-\sqrt{6}} \times \frac{5+\sqrt{6}}{5+6} = \frac{19\sqrt{2}+19\sqrt{3}}{19} = \sqrt{2}+\sqrt{3}$$

$$\frac{1}{-(\sqrt{3}-1)} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{-\sqrt{3}+1}{2} \rightarrow \boxed{\sqrt{2}-1}$$

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ب)

$$\frac{3\sqrt{3}-1}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}} = \frac{12\sqrt{3}-9-4+\sqrt{3}}{13}$$

$$= \frac{13(\sqrt{3}-1)}{13} + (2+\sqrt{3}) = \boxed{1+2\sqrt{3}}$$

۴) الف)

$$\frac{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}}-2=$$

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$$\left(\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}\right)^{2}=1+\sqrt{3}+\sqrt{3}-1+2\sqrt{3-1}$$

$$=\sqrt{2\sqrt{3}+2\sqrt{2}}=\sqrt{2\times(\sqrt{3}+\sqrt{2})}=\sqrt{2}\times\sqrt{\sqrt{3}+\sqrt{2}}$$

$$\Rightarrow\frac{\sqrt{2(\sqrt{3}+\sqrt{2})}}{\sqrt{\sqrt{3}-\sqrt{2}}}\times\frac{\sqrt{\sqrt{3}+\sqrt{2}}}{\sqrt{\sqrt{3}+\sqrt{2}}}=\frac{2+\sqrt{6}}{1},2$$

$$=\boxed{\sqrt{6}}$$

ب)

$$2^{\frac{8}{12}}\times 3\times 2^{\frac{1}{4}}\times 2^{\frac{13}{12}}=3\times 2^{2}=12$$

۵)

$$\frac{3^{n}(1+3+9+27+81+243)}{2^{n-2}(1+2+4+8+16+32)}=\frac{3^{n}}{2^{n-2}}\times\frac{364}{63}$$

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$$=\frac{52}{9}\times\frac{3^{n}}{2^{n-2}}=\frac{52}{9}\times 4\times\left(\frac{3}{2}\right)^{n}=52$$

$$\frac{4}{9}=\left(\frac{2}{3}\right)^{n}\Rightarrow\boxed{n=2}$$

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$$(a^2+b^2-2ab)^2\,(a^2+b^2+2ab)^2 =$$

$$(a^2+b^2)^2-(2ab)^2 = (a^2-b^2)^2 =$$

$$a^2=\sqrt{\sqrt{6}-2} \qquad b=\sqrt{\sqrt{6}+2}$$

$$(b^2-a^2)^2 = \left(\sqrt{\sqrt{6}+2}-\sqrt{\sqrt{6}-2}\right)^2 = 2\sqrt{6}-2\sqrt{(\sqrt{6}+2)(\sqrt{6}-2)}$$

$$= 2\sqrt{6}-2\sqrt{2} =$$

$$(a^2-b^2)^2 = (2\sqrt{6}-2\sqrt{2})^2 = 4(6+2-2\sqrt{12})$$

$$= \boxed{32-16\sqrt{3}} =$$

۷)

$$\left(a^2+\frac{1}{a^2}\right)^2 \qquad a=\sqrt[4]{\underbrace{7-4\sqrt{3}}_{(2-\sqrt{3})^2}}$$

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$$a^2=2-\sqrt{3} \qquad \frac{1}{a^2}=2+\sqrt{3}$$

$$a^2+\frac{1}{a^2}=2-\cancel{\sqrt{3}}+2+\cancel{\sqrt{3}}=4$$

$$2^t=4^2=16 \Rightarrow \boxed{t=4}$$

۸)

$\sqrt[۵]{۲^{۲}\sqrt[۳]{\cancel{۱۶}}}$ $\quad (\frac{۱}{۲})^{-\frac{۴}{۳}} = \sqrt[۱۵]{\cancel{۲^{۱۵۲}}} \times ۲^{\frac{۴}{۳}}$

$= ۲^{\frac{۶}{۳}} = ۲^{۲} \Rightarrow (۲A)^{-\frac{۱}{۳}} = \frac{۱}{(۲^{۳})^{\frac{۱}{۳}}} = \boxed{\frac{۱}{۲}}$

۹)

$\sqrt[۷]{a} = \sqrt{۲۷ a^{\frac{۱۵}{۷}}}$

$a^{\frac{۱}{۷}-\frac{۱۵}{۷}} = ۲۷ \Rightarrow a^{-۲} = ۲۷ \Rightarrow a^{۲} = \frac{۱}{۲۷}$

$\Rightarrow a = \frac{۱}{۳\sqrt{۳}} \qquad \frac{۱}{a} = ۳\sqrt{۳} \qquad \frac{۱}{a} - ۳ = ۳(\sqrt{۳}-۱)$

$(۱+\sqrt{۳})(\frac{۱}{a}-۳) = ۳(۱+\sqrt{۳})(\sqrt{۳}-۱) = \boxed{۶}$

۱۰)

$\sqrt{x+a} - \sqrt{x-۴} = ۲$

$(\sqrt{x+a}-\sqrt{x-۴})(\sqrt{x+a}+\sqrt{x-۴}) = x+a-(x-۴)$

$= a+۴$

$\cancel{۲}(\sqrt{x+a}+\sqrt{x-۴})^{-۲} = \frac{a+۴}{۲} - ۲ = \boxed{\frac{a}{۲}}$