۱) عامل $۵ \times ۲ \leftarrow ۶! \,,\, ۵!$

$۱! = ۱ \quad ۳! = ۶ \rightarrow ۱+۶=۷$

$۲! = ۲ \quad ۴! = ۲۴ \rightarrow ۲+۲۴ = ۲۶$

$۶ \times ۷ = ۴۲ \rightarrow$ یکان

۲) الف)

$$\frac{۱}{\sqrt[۴]{۴+\sqrt{۷}}} \times \frac{\sqrt[۴]{(۱+\sqrt{۷})^۲}}{۱} \quad (۸+۲\sqrt{۷})$$

$$= \frac{\sqrt[۴]{۲(۴+\sqrt{۷})}}{\sqrt[۴]{۴+\sqrt{۷}}} = \boxed{\sqrt[۴]{۲}}$$

ب)

$$\frac{\sqrt{۶-۲\sqrt{۵}}}{\sqrt{۲}} = \frac{\sqrt{۲} \times \sqrt{۳-\sqrt{۵}}}{\sqrt{۲}} \qquad \sqrt{(۱-\sqrt{۵})^۲}$$

$$\frac{\sqrt{\cancel{۳}+\sqrt{۵}}}{\sqrt{۲}\,(\cancel{\sqrt{۵}+\sqrt{۲}})}$$

$$\frac{\sqrt{۲} \times -\sqrt{۳+\sqrt{۵}}}{\sqrt{۲}} = \frac{-(۱+\sqrt{۵})}{\sqrt{۲}} \qquad = \frac{-۲}{۲} = -۱$$

۳) الف)

$$\frac{۲\sqrt{۲}+۳\sqrt{۳}}{۵-\sqrt{۶}} \times \frac{۵+\sqrt{۶}}{۵+\sqrt{۶}} = \frac{۱۹\sqrt{۲}+۱۹\sqrt{۳}}{۱۹} = \sqrt{۲}+\sqrt{۳}$$

$$\frac{۱}{-(\sqrt{۳}-۱)} \times \frac{\sqrt{۳}+۱}{\sqrt{۳}+۱} = \frac{-\sqrt{۳}+۱}{۲} \rightarrow \boxed{\sqrt{۲}-۱}$$

ب)

$$\frac{۳\sqrt{۳}-۱}{۴+\sqrt{۳}} \times \frac{۴-\sqrt{۳}}{۴-\sqrt{۳}} = \frac{۱۲\sqrt{۳}-۹-۴+\sqrt{۳}}{۱۳}$$

$$= \frac{\cancel{۱۳}(\sqrt{۳}-۱)}{\cancel{۱۳}} + (۲+\sqrt{۳}) = \boxed{۱+۲\sqrt{۳}}$$

۴) الف)

$$\frac{\sqrt{۱+\sqrt{۳}}+\sqrt{\sqrt{۳}-۱}}{\sqrt{\sqrt{۳}-\sqrt{۲}}}-۲=$$

$$\left(\sqrt{۱+\sqrt{۳}}+\sqrt{\sqrt{۳}-۱}\right)^{۲}=۱+\sqrt{۳}+\sqrt{۳}-۱+۲\sqrt{۳-۱}$$

$$=\sqrt{۲\sqrt{۳}+۲\sqrt{۴}}=\sqrt{۲\times(\sqrt{۳}+\sqrt{۲})}=\sqrt{۲}\times\sqrt{\sqrt{۳}+\sqrt{۲}}$$

$$\Rightarrow\frac{\sqrt{۲(\sqrt{۳}+\sqrt{۲})}}{\sqrt{\sqrt{۳}-\sqrt{۲}}}\times\frac{\sqrt{\sqrt{۳}+\sqrt{۲}}}{\sqrt{\sqrt{۳}+\sqrt{۲}}}=\frac{۲+\sqrt{۶}}{۱},۲$$

$$=\boxed{\sqrt{۶}}$$

ب)

$$۲^{\frac{۸}{۱۲}}\times۳\times۲^{\frac{۱}{۴}}\times۲^{\frac{۱۳}{۱۲}}=۳\times۲^{۲}=۱۲$$

۵)

$$\frac{۳^{n}(۱+۳+۹+۲۷+۸۱+۲۴۳)}{۲^{n-۲}(۱+۲+۴+۸+۱۶+۳۲)}=\frac{۳^{n}}{۲^{n-۲}}\times\frac{۳۶۴}{۶۳}$$

$$=\frac{۵۲}{۹}\times\frac{۳^{n}}{۲^{n-۲}}=\frac{۵۲}{۹}\times۴\times\left(\frac{۳}{۲}\right)^{n}=۵۲$$

$$\frac{۴}{۹}=\left(\frac{۲}{۳}\right)^{n}\Rightarrow\boxed{n=۲}$$

(۶

$$(a^2+b^2-2ab)^2\,(a^2+b^2+2ab)^2 =$$

$$(a^2+b^2)^2-(2ab)^2 = (a^2-b^2)^2 =$$

$a^2 = \sqrt{\sqrt{6}-2}$ $\quad b = \sqrt{\sqrt{6}+2}$

$$(b^2-a^2)^2 = \left(\sqrt{\sqrt{6}+2}-\sqrt{\sqrt{6}-2}\right)^2 = 2\sqrt{6}-2\sqrt{(\sqrt{6}+2)(\sqrt{6}-2)}$$

$$= 2\sqrt{6}-2\sqrt{2} =$$

$$(a^2-b^2)^2 = (2\sqrt{6}-2\sqrt{2})^2 = 4(6+2-2\sqrt{12})$$

$$= \boxed{32-16\sqrt{3}} =$$

(۷

$$\left(a^2+\frac{1}{a^2}\right)^2 \qquad a = \sqrt[4]{\underbrace{7-4\sqrt{3}}_{(2-\sqrt{3})^2}}$$

$a^2 = 2-\sqrt{3}$ $\quad \frac{1}{a^2} = 2+\sqrt{3}$

$$a^2+\frac{1}{a^2} = 2-\cancel{\sqrt{3}}+2+\cancel{\sqrt{3}} = 4$$

$$2^t = 4^2 = 16 \Longrightarrow \boxed{t=4}$$

۸)

$$\sqrt[5]{2^2\sqrt[3]{\cancel{16}}}\left(\frac{1}{2}\right)^{-\frac{4}{3}} = \sqrt[15]{\cancel{2}^{102}} \times 2^{\frac{4}{3}}$$

$$= 2^{\frac{6}{3}} = 2^2 \Rightarrow (2A)^{-\frac{1}{3}} = \frac{1}{(2^3)^{\frac{1}{3}}} = \boxed{\frac{1}{2}}$$

۹)

$$\sqrt[7]{a} = \sqrt{27a^{\frac{15}{7}}}$$

$$a^{\frac{1}{7}-\frac{15}{7}} = 27 \Rightarrow a^{-2} = 27 \Rightarrow a^2 = \frac{1}{27}$$

$$\Rightarrow a = \frac{1}{3\sqrt{3}} \qquad \frac{1}{a} = 3\sqrt{3} \qquad \frac{1}{a} - 3 = 3(\sqrt{3}-1)$$

$$(1+\sqrt{3})\left(\frac{1}{a}-3\right) = 3(1+\sqrt{3})(\sqrt{3}-1) = \boxed{6}$$

۱۰)

$$\sqrt{x+a} - \sqrt{x-4} = 2$$

$$(\sqrt{x+a} - \sqrt{x-4})(\sqrt{x+a} + \sqrt{x-4}) = x+a-(x-4)$$

$$= a+4$$

$$\cancel{2}(\sqrt{x+a} + \sqrt{x-4})^{-2} = \frac{a+4}{2} - 2 = \boxed{\frac{a}{2}}$$