

رقم یکان عدد $(1! + 3! + 5! + \dots + 25!)(2! + 4! + 6! + \dots + 26!)$ را بیابید.

$$\begin{array}{l} 1! \rightarrow 1 \\ 3! \rightarrow 9 \\ 5! \rightarrow 120 \\ \vdots \\ 25! \rightarrow \text{یکان صفر} \end{array} \quad \begin{array}{l} 2! \rightarrow 2 \\ 4! \rightarrow 24 \\ 6! \rightarrow 720 \\ \vdots \\ 26! \rightarrow \text{یکان صفر} \end{array}$$

(۱)

الف) $\sqrt[4]{(4+\sqrt{7})^{-1}} \sqrt[4]{(1+\sqrt{7})^2} \rightarrow \frac{1}{\sqrt[4]{4+\sqrt{7}}} \times \sqrt[4]{(1+\sqrt{7})^2} \rightarrow \sqrt[4]{\frac{(1+\sqrt{7})^2}{4+\sqrt{7}}}$

(۲)

ب) $\left(\frac{\sqrt{2}+\sqrt{5}}{\sqrt{10}+\sqrt{2}}\right) \left(\sqrt{3}-\sqrt{5}-\sqrt{3+\sqrt{5}}\right)$

$$\rightarrow \frac{1}{\sqrt{2}} \left(\sqrt{3}-\sqrt{5}-\sqrt{3+\sqrt{5}}\right) \rightarrow \frac{1}{\sqrt{2}} \left(\sqrt{3}-\sqrt{5}-\sqrt{3+\sqrt{5}}\right)$$

(۳)

الف) $\frac{\sqrt{18}+\sqrt{27}}{5-\sqrt{6}} - 2(\sqrt{4}-1)^{-1} \rightarrow \frac{3\sqrt{2}+3\sqrt{3}}{5-\sqrt{6}} \times \frac{5+\sqrt{6}}{5+\sqrt{6}} = \frac{15\sqrt{2}+15\sqrt{3}+3\sqrt{2}+3\sqrt{3}}{19} = \frac{18\sqrt{2}+18\sqrt{3}}{19}$

ب) $\frac{\sqrt{27}-1}{4+\sqrt{3}} + (2-\sqrt{3})^{-1} \rightarrow \sqrt{3}-1 + \frac{1}{2-\sqrt{3}} = \sqrt{3}-1 + \frac{2+\sqrt{3}}{1} = \sqrt{3}+\sqrt{3}+1$

(۴)

الف) $\frac{(\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1})^2}{\sqrt{\sqrt{3}-\sqrt{2}}} - 2 \rightarrow \frac{1+\sqrt{3}+\sqrt{3}-1+2\sqrt{3}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{2\sqrt{3}+2\sqrt{2}}{1} = 2(\sqrt{3}+\sqrt{2})$

ب) $\sqrt[4]{2\sqrt{2}} \times \sqrt[4]{162} \times \sqrt[4]{4\sqrt{2}} \times \sqrt[4]{2} \times \sqrt[4]{2} \times \sqrt[4]{2} \rightarrow 2^{\frac{1}{4}} \times 2^{\frac{3}{4}} \times 2^{\frac{1}{4}} \times 2^{\frac{1}{4}} \times 2^{\frac{1}{4}} \times 2^{\frac{1}{4}} = 2^{\frac{6}{4}} = 2^{\frac{3}{2}} = 2\sqrt{2}$

(۵)

$\frac{3^x+3^{x+1}+3^{x+2}+3^{x+3}+3^{x+4}+3^{x+5}}{3^x-2+3^{x-1}+3^x+3^{x+1}+3^{x+2}+3^{x+3}} = 52$

(۶)

$(a^2+b^2-2ab)^t (a^2+b^2+2ab)^t = t(2-\sqrt{3})$

$$(a-b)^{2t} (a+b)^{2t} = t(2-\sqrt{3})$$

(۷)

$\left(a + \frac{1}{a} + \sqrt{2}\right)^t \left(a + \frac{1}{a} - \sqrt{2}\right)^t = 2^t$

$$\left(a + \frac{1}{a}\right)^t - (\sqrt{2})^t = 2^t$$

(۸) اگر $A = \sqrt[3]{4 \sqrt[3]{16}} \left(\frac{1}{2}\right)^{-\frac{2}{3}}$ باشد، حاصل $(2A)^{-\frac{1}{3}}$ را بیابید.

$(1)^{-\frac{1}{3}} = (4^{\frac{2}{3}})^{-\frac{1}{3}} = \frac{1}{4^{\frac{2}{9}}}$
 $\sqrt[3]{4^{\frac{2}{3}}} \rightarrow 4^{\frac{2}{9}} \cdot \frac{1}{2} = 4^{\frac{2}{9}} \cdot 2^{-\frac{1}{3}} = 2^{\frac{4}{9} - \frac{2}{3}} = 2^{-\frac{2}{9}}$
 $\downarrow$
 $a = 2^{\frac{2}{9}} \cdot 2^{\frac{2}{9}}$

(۹) $\sqrt[n]{a} \cdot \sqrt[n]{a} \frac{1}{\sqrt[n]{a}} \rightarrow a = \sqrt[n]{a} \frac{1}{\sqrt[n]{a}} \rightarrow a^{\frac{1}{n}} \cdot a^{-\frac{1}{n}}$

$(a^{\frac{1}{n}})^{\frac{1}{n}} (a^{-\frac{1}{n}})^{\frac{1}{n}} \rightarrow a^{\frac{1}{n^2}} a^{-\frac{1}{n^2}} = a^{-\frac{1}{n^2}} \rightarrow \frac{1}{a^{\frac{1}{n^2}}} = \frac{1}{\sqrt[n^2]{a}}$
 $\frac{1}{\sqrt[n^2]{a}} = \frac{1}{\sqrt[n^2]{a}} \cdot \frac{1}{\sqrt[n^2]{a}} = \frac{1}{\sqrt[n^2]{a^2}} = \frac{1}{\sqrt[n^2]{4 \cdot 16}} = \frac{1}{\sqrt[n^2]{64}} = \frac{1}{4}$

(۱۰) $\sqrt{x+a} + \sqrt{x-a} = 2$

$\sqrt{x+a} = 2 - \sqrt{x-a}$
 $\sqrt{x+a} = 2 - \sqrt{x-a}$
 $\sqrt{x+a} = 2 - \sqrt{x-a}$

$A + B = 2$
 $A - B = 2$
 $2A = 4 \rightarrow A = 2$
 $2B = 0 \rightarrow B = 0$

$A - B = 2$
 $(A + B)^2 = 4$
 $A^2 + 2AB + B^2 = 4$
 $4 + 2AB + 0 = 4$
 $2AB = 0 \rightarrow AB = 0$