الله دادخواه ۵ اعجلی ها

۱-

$$\left(1!+3!+5!+\dots\right)\left(2!+4!+\dots\right)=7\times 6=4\textcircled{2}$$ → رقم یکان = ۲

($\overbrace{1!+3!}^{6+1}$ ، $\overbrace{2!+4!}^{2+4=6}$)

۲-

الف) $\sqrt[4]{(4+\sqrt{7})^{-1}(1+\sqrt{7})^{2}}=\sqrt[4]{\frac{8+2\sqrt{7}}{4+\sqrt{7}}}=\sqrt[4]{\frac{2(4+\sqrt{7})}{4+\sqrt{7}}}=\sqrt[4]{2}\rightarrow 2^{\frac{1}{4}}$

ب) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{10}+2}=\frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}(\sqrt{5}+\sqrt{2})}=\frac{1}{\sqrt{2}}$

$\sqrt{3-\sqrt{5}}-\sqrt{3+\sqrt{5}}=\sqrt{\frac{(1-\sqrt{5})^2}{2}}-\sqrt{\frac{(1+\sqrt{5})^2}{2}}=\frac{\sqrt{(1-\sqrt{5})^2}}{\sqrt{2}}-\frac{\sqrt{(1+\sqrt{5})^2}}{2}=\frac{\sqrt{5}-1}{\sqrt{2}}-\frac{1+\sqrt{5}}{\sqrt{2}}=\frac{-2}{\sqrt{2}}$

$\longrightarrow \frac{1}{\sqrt{2}}\times\frac{-2}{\sqrt{2}}=\frac{-2}{2}=-1$

۳-

الف) $\underbrace{\frac{\sqrt{8}+\sqrt{27}}{5-\sqrt{6}}}_{T}-\underbrace{\frac{2}{\sqrt{3}-1}}_{S}$

$\rightarrow T=\frac{\sqrt{8}+\sqrt{27}}{5-\sqrt{6}}\times\frac{(5+\sqrt{6})}{(5+\sqrt{6})}=\frac{5\sqrt{8}+5\sqrt{27}+\sqrt{48}+\sqrt{162}}{25-6}=\frac{10\sqrt{2}+15\sqrt{3}+4\sqrt{3}+9\sqrt{2}}{19}\rightarrow\sqrt{3}+\sqrt{2}$

$\rightarrow S=\frac{2}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}=\frac{\sqrt{3}+1}{1}\longrightarrow T-S=\sqrt{2}-1$

ب) $\underbrace{\frac{\sqrt{27}-1}{4+\sqrt{3}}}_{M}-\underbrace{\frac{1}{2-\sqrt{3}}}_{N}$

$\rightarrow M=\frac{3\sqrt{3}-1}{4+\sqrt{3}}\times\frac{(4-\sqrt{3})}{(4-\sqrt{3})}=\frac{12\sqrt{3}-4-9+\sqrt{3}}{13}=\sqrt{3}-1$

$\rightarrow N=\frac{1}{2-\sqrt{3}}\times\frac{(2+\sqrt{3})}{(2+\sqrt{3})}=2+\sqrt{3}\longrightarrow M-N=1+2\sqrt{3}$

۴-

الف) $\frac{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}}-2=$

$\left(\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}\right)^2=2\sqrt{3}+2\sqrt{2}\xrightarrow{\text{پس}}\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}=\sqrt{2\sqrt{3}+2\sqrt{2}}$

$\sqrt{\frac{2\sqrt{3}+2\sqrt{2}}{\sqrt{3}-\sqrt{2}}\times\frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})}}=\frac{2\sqrt{3}+2\sqrt{2}\times(\sqrt{3}+\sqrt{2})}{1}\longrightarrow 10+4\sqrt{6}\rightarrow\sqrt{10+4\sqrt{6}}=\sqrt{(\sqrt{6}+2)^2}=\sqrt{6}+2$

$\sqrt{6}+2-2=\sqrt{6}$

ب) $\sqrt[12]{2^8}\times\sqrt[4]{81\times 2}\times\sqrt[12]{3^{12}\times 2}=\sqrt[12]{2^{21}}\times 3\sqrt[4]{2}=3\sqrt[4]{2^8}=3\times 4=12$

$\sqrt[4]{81\times 2}\rightarrow 3\sqrt[4]{2}$ ، $\sqrt[12]{3^{12}\times 2}\rightarrow 3\sqrt[12]{2}$ ، $\sqrt[12]{2^{21}}\rightarrow\sqrt[4]{2^7}$

۵-

$$\frac{3^x\left(\cancel{1+3^1+3^2+3^3+3^4+3^5}\right)}{2^{x-2}\left(\cancel{1+2+2^2+2^3+2^4+2^5}\right)}=52\longrightarrow\frac{3^x}{2^{x-2}}\times 364=63\times 52\rightarrow\frac{3^x}{2^{x-2}}=9$$

(۳۶۴ ، ۶۳)

$\longrightarrow\boxed{x=2}$ به ازای ۲ جواب می‌دهد

۶-

$(a-b)^{۴} \times (a+b)^{۴} = (a^{۲}-b^{۲})^{۴}$

$۱۶ \quad \cancel{۸} ۲(۲-\sqrt{۳})$

$\left(\sqrt{\sqrt{۶}+۲} - \sqrt{\sqrt{۶}-۲}\right)^{۴} = \left(\sqrt{۶}+۲+\sqrt{۶}-۲-۲\sqrt{۲}\right)^{۲} = ۴(۶+۲-۴\sqrt{۳}) = ۸(۴-۲\sqrt{۳}) =$

$۱۶(۲-\sqrt{۳}) \to ۱۶ = t$

۷-

$\left((a+\frac{۱}{a})^{۲}-۲\right)^{۲} = \left(a^{۲}+\frac{۱}{a^{۲}}+۲-۲\right)^{۲} = a^{۴}+\frac{۱}{a^{۴}}+۲$

$= ۷-۴\sqrt{۳} + \frac{۱}{۷-۴\sqrt{۳}} + ۲ = ۹-۴\sqrt{۳} + \frac{۷+۴\sqrt{۳}}{۴۹-۴۸} = ۱۶$

۸-

$A = \sqrt[۵]{۴\sqrt[۳]{۱۶}}\left(\frac{۱}{۲}\right)^{-\frac{۴}{۳}} = \sqrt[۵]{۲^{۲}\times ۲^{\frac{۴}{۳}}} \times ۲^{\frac{۴}{۳}} = \sqrt[۵]{۲^{\frac{۱۰}{۳}}} \times ۲^{\frac{۴}{۳}} = ۲^{\frac{۲}{۳}} \times ۲^{\frac{۴}{۳}} = ۲^{۲}$

$(۲A)^{-\frac{۱}{۳}} = ۸^{-\frac{۱}{۳}} = (۲^{۳})^{-\frac{۱}{۳}} = ۲^{-۱} = \frac{۱}{۲}$

۹-

$\sqrt[۷]{a} = ۲۷ \times a^{\frac{۱۵}{۷}} \xrightarrow{\div \sqrt[۷]{a}} ۱ = ۲۷ \times \frac{a^{\frac{۱۵}{۷}}}{a^{\frac{۱}{۷}}} \Rightarrow ۱ = ۲۷a^{۲} \to a^{۲} = \frac{۱}{۲۷} \to a = \frac{۱}{۳\sqrt{۳}}$

$\frac{(\frac{۱}{a}-۳)}{۱+\sqrt{۳}} = \frac{۳\sqrt{۳}-۳}{۱+\sqrt{۳}} \times \frac{۱-\sqrt{۳}}{۱-\sqrt{۳}} = \frac{۳\sqrt{۳}-۹-۳+۳\sqrt{۳}}{(۱-۳} = \frac{۶\sqrt{۳}-۱۲}{-۲} = -۳\sqrt{۳}+۶$

۱۰-

$\underbrace{\sqrt{x+۹}}_{M} - \underbrace{\sqrt{x-۴}}_{N} = ۲ \qquad \sqrt{x+۹} + \sqrt{x-۴} \to$

$M+N-(M-N) = ۲N = ۲(\sqrt{x-۴})$

$M-N = ۲$