Subject : Date: ۴۵

پرنیان باقری / تکلیف شماره ۲۱ / کلاس دهم دختر A

۱

$\underbrace{(۱!+۳!+۵!+\dots+۲۵!)}\underbrace{(۲!+۴!+۶!+\dots+۲۶!)}$

$۱!+۳!=\boxed{۷}$ $\qquad$ $۲!+۴!=۲\boxed{۶}\rightarrow$ یکان

$\Longrightarrow ۷\times۶=۴۲\rightarrow$ یکان

جواب نهایی

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الف) $\sqrt[۴]{(۴+\sqrt{۷})^{-۱}}\times\sqrt{۱+\sqrt{۷}}=\sqrt[۴]{(۴+\sqrt{۷})^{-۱}}\times\sqrt[۴]{(۱+\sqrt{۷})^{۲}}=$

$\sqrt[۴]{(۴+\sqrt{۷})^{-۱}}\times\sqrt[۴]{۸+۲\sqrt{۷}}=\sqrt[۴]{\dfrac{۸+۲\sqrt{۷}}{۴+\sqrt{۷}}}=\boxed{\sqrt[۴]{۲}}$

ب) $\left(\dfrac{\sqrt{۲}+\sqrt{۵}}{\sqrt{۱۰}+۲}\right)\left(\sqrt{۳-\sqrt{۵}}-\sqrt{۳+\sqrt{۵}}\right)=$

$\left(\dfrac{\cancel{\sqrt{۲}+\sqrt{۵}}}{\sqrt{۲}(\cancel{\sqrt{۵}+\sqrt{۲}})}\right)\left(\sqrt{۳-\sqrt{۵}}-\sqrt{۳+\sqrt{۵}}\right)=\dfrac{\sqrt{۳-\sqrt{۵}}-\sqrt{۳+\sqrt{۵}}}{\sqrt{۲}}\times\dfrac{\sqrt{۲}}{\sqrt{۲}}=$

$\dfrac{\sqrt{۶-۲\sqrt{۵}}-\sqrt{۶+۲\sqrt{۵}}}{۲}=\dfrac{\sqrt{(\sqrt{۵}-۱)^{۲}}-\sqrt{(\sqrt{۵}+۱)^{۲}}}{۲}=\dfrac{\sqrt{۵}-۱-\sqrt{۵}-۱}{۲}=\boxed{(-۱)}$

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الف) $\dfrac{\sqrt{۸}+\sqrt{۲۷}}{۵-\sqrt{۶}}-۲(\sqrt[۴]{۹}-۱)^{-۱}=\dfrac{۲\sqrt{۲}+۳\sqrt{۳}}{(\sqrt{۲})^{۲}+(\sqrt{۳})^{۲}-\sqrt{۳}\times\sqrt{۲}}-\dfrac{۲}{\sqrt{۳}-۱}=$

$\dfrac{(۲\sqrt{۲}+۳\sqrt{۳})(\sqrt{۲}+\sqrt{۳})}{\underbrace{((\sqrt{۲})^{۲}+(\sqrt{۳})^{۲}-\sqrt{۳}\times\sqrt{۲})(\sqrt{۲}+\sqrt{۳})}_{۲\sqrt{۲}+۳\sqrt{۳}}}-\dfrac{\cancel{۲}(\sqrt{۳}+۱)}{\frac{۳-۱}{\cancel{۲}}}=\sqrt{۲}+\sqrt{۳}-\sqrt{۳}-۱=\boxed{\sqrt{۲}-۱}$

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ب) $\frac{\sqrt{27}-1}{4+\sqrt{3}} + (2-\sqrt{3})^{-1} = \frac{3\sqrt{3}-1}{1+3-\sqrt{3}} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} + \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} =$

$\frac{(\cancel{3\sqrt{3}-1})(\sqrt{3}-1)}{\cancel{3\sqrt{3}-1}} + 2+\sqrt{3} = \sqrt{3}-1+2+\sqrt{3} = \boxed{2\sqrt{3}+1}$

4 الف) $\frac{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}} - 2$

$\frac{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}} \times \frac{\sqrt{\sqrt{3}+\sqrt{2}}}{\sqrt{\sqrt{3}+\sqrt{2}}} = (\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1})(\sqrt{\sqrt{3}-\sqrt{2}}) =$

$\sqrt{\left[(\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1})(\sqrt{\sqrt{3}-\sqrt{2}})\right]^2} = \sqrt{(2\sqrt{3}-2\sqrt{2})(\sqrt{3}-\sqrt{2})} =$

$(\sqrt{3}-\sqrt{2})\sqrt{2} \Longrightarrow \sqrt{2}(\sqrt{3}-\sqrt{2})-2 = \boxed{\sqrt{6}-4}$

ب) $\sqrt[4]{\sqrt[3]{2^8}} \times \sqrt[4]{162} \times \sqrt{4\sqrt[6]{2}} = 2^{\frac{2}{3}} \times 2^{\frac{1}{4}} \times 3 \times 2 \times 2^{\frac{1}{12}} = 3 \times 4 = \boxed{12}$

5 $\frac{3^x+3^{x+1}+3^{x+2}+3^{x+3}+3^{x+4}+3^{x+5}}{2^{x-2}+2^{x-1}+2^{x}+2^{x+1}+2^{x+2}+2^{x+3}} = 52$

$\frac{3^x(1+3+9\cdots+3^5)}{2^{x-2}(1+2+4+\cdots+2^5)} = \frac{3^x\left(\frac{3^6-1}{2}\right)}{2^{x-2}(2^6-1)} = \frac{3^x(\overset{26\times28}{3^6-1})}{2\times2^{x-2}(\underset{63}{2^6-1})} =$

$\frac{\overset{3^{x-2}}{\cancel{3^x}} \times \overset{4}{\cancel{28}} \times \overset{2}{\cancel{26}}}{\underset{2^{x-2}}{\cancel{2^{x-1}}} \times \underset{9}{\cancel{63}}} = \frac{\cancel{52}}{\cancel{13\times2^2}} \Longrightarrow \frac{3^{x-2}}{2^{x-2}} = 1 \Rightarrow \boxed{x=2}$

۶

$$(a^۲+b^۲-۲ab)^۲(a^۲+b^۲+۲ab)^۲=((a^۲+b^۲)^۲-۴a^۲b^۲)^۲$$

$$(a^۴+b^۴+۲a^۲b^۲-۴a^۲b^۲)^۲=(a^۴+b^۴-۲a^۲b^۲)^۲=$$

$$\Big[\underbrace{(\sqrt[۴]{\sqrt{۶}-۲})^۴+(\sqrt[۴]{\sqrt{۶}+۲})^۴}_{۲\sqrt{۶}}-۲\underbrace{(\sqrt{\sqrt{۶}-۲})(\sqrt{\sqrt{۶}+۲})}_{\sqrt{۲}}\Big]^۲=$$

$$(۲\sqrt{۶}-۲\sqrt{۲})^۲=[۲\sqrt{۲}(\sqrt{۳}-۱)]^۲=۱۶(۲-\sqrt{۳})=t(۲-\sqrt{۳})$$

$$\Rightarrow \boxed{t=۱۶}$$

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$$a=\sqrt[۴]{۷-۴\sqrt{۳}}=\sqrt{(۲-\sqrt{۳})^۲}=\sqrt{۲-\sqrt{۳}}$$

$$(a+\frac{۱}{a}+\sqrt{۲})^۲(a+\frac{۱}{a}-\sqrt{۲})^۲=((a+\frac{۱}{a})^۲-۲)^۲=$$

$$(a^۲+\frac{۱}{a^۲}+۲-۲)^۲=(a^۲+\frac{۱}{a^۲})^۲=(۲-\sqrt{۳}+\frac{۱}{۲-\sqrt{۳}})^۲=$$

$$(۲-\sqrt{۳}+۲+\sqrt{۳})^۲=۱۶=۲^t \Rightarrow \boxed{t=۴}$$

۸

$$A=\sqrt[۵]{۴\sqrt[۳]{۱۶}}\times(\frac{۱}{۲})^{-\frac{۴}{۳}}=۲^{\frac{۲}{۳}}\times۲^{\frac{۴}{۳}}=۴$$

$$(۲A)^{-\frac{۱}{۳}}=۸^{-\frac{۱}{۳}}=\boxed{\frac{۱}{۲}}$$

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$$27a^{\frac{15}{7}} = a^{\frac{1}{7}} \Rightarrow 27 \times a^{2} = 1 \Rightarrow a = \frac{1}{3\sqrt{3}}$$

$$\frac{\frac{1}{a} - 3}{1+\sqrt{3}} = \frac{3\sqrt{3} - 3}{1+\sqrt{3}} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{3(4-2\sqrt{3})}{2} = \boxed{6-3\sqrt{3}}$$

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$$(\underbrace{\cancel{\sqrt{x+a} - \sqrt{x-4}}}_{2})(\sqrt{x+a} + \sqrt{x-4}) = a+4$$

$$\sqrt{x+a} + \sqrt{x-4} = \frac{a+4}{2}$$

$$\sqrt{x+a} + \sqrt{x-4} - 2 = \frac{a}{2} + 2 - 2 = \boxed{\frac{a}{2}}$$