

پرشیان باقری / تالیف شماره ۲۱ / لاس دهم دختر A

$$1 \quad (1! + 3! + 5! + \dots + 25!) (2! + 4! + 6! + \dots + 24!)$$

$$1! + 3! = \boxed{4}$$

$$2! + 4! = 2 \times 4 = 8$$

$$\Rightarrow 4 \times 8 = 32 \rightarrow \text{پاکان}$$

جواب نهایی

$$2 \quad \text{الف) } \sqrt[4]{(4+\sqrt{7})^{-1}} \times \sqrt{1+\sqrt{7}} = \sqrt[4]{(4+\sqrt{7})^{-1}} \times \sqrt[4]{(1+\sqrt{7})^2}$$

$$\sqrt[4]{(4+\sqrt{7})^{-1}} \times \sqrt[4]{1+2\sqrt{7}} = \sqrt[4]{\frac{1+2\sqrt{7}}{4+\sqrt{7}}} = \boxed{\sqrt[4]{2}}$$

$$\text{ب) } \left(\frac{\sqrt{2}+\sqrt{5}}{\sqrt{5}+2} \right) (\sqrt{3}-\sqrt{5} - \sqrt{3+\sqrt{5}}) =$$

$$\left(\frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}(\sqrt{5}+\sqrt{2})} \right) (\sqrt{3}-\sqrt{5} - \sqrt{3+\sqrt{5}}) = \frac{\sqrt{3}-\sqrt{5} - \sqrt{3+\sqrt{5}}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} =$$

$$\frac{\sqrt{4-2\sqrt{5}} - \sqrt{4+2\sqrt{5}}}{2} = \frac{\sqrt{(\sqrt{5}-1)^2} - \sqrt{(\sqrt{5}+1)^2}}{2} = \frac{\sqrt{5}-1 - \sqrt{5}-1}{2} = \boxed{-1}$$

$$3 \quad \text{الف) } \frac{\sqrt{8}+\sqrt{27}}{5-\sqrt{4}} - 2(\sqrt[4]{9}-1)^{-1} = \frac{2\sqrt{2}+3\sqrt{3}}{(\sqrt{2})^2+(\sqrt{3})^2-\sqrt{3}\sqrt{2}} - \frac{2}{\sqrt{3}-1} =$$

$$\frac{(2\sqrt{2}+3\sqrt{3})(\sqrt{2}+\sqrt{3})}{((\sqrt{2})^2+(\sqrt{3})^2-\sqrt{3}\sqrt{2})(\sqrt{2}+\sqrt{3})} \times \frac{\sqrt{3}+1}{\sqrt{2}-1} = \frac{\sqrt{2}+\sqrt{3}-\sqrt{3}-1}{\sqrt{2}-1} =$$

$2\sqrt{2}+3\sqrt{3}$

$$ب) \frac{\sqrt{3}-1}{1+\sqrt{3}} + (1-\sqrt{3})^{-1} = \frac{\sqrt{3}-1}{1+\sqrt{3}} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} + \frac{1}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} =$$

$$\frac{(\sqrt{3}-1)(\sqrt{3}-1)}{1+\sqrt{3}-\sqrt{3}-1} + \frac{1+\sqrt{3}}{1-\sqrt{3}} = \frac{\sqrt{3}-1+\sqrt{3}-1}{1+\sqrt{3}-\sqrt{3}-1} + \frac{1+\sqrt{3}}{1-\sqrt{3}} = \frac{2\sqrt{3}-2}{0} + \frac{1+\sqrt{3}}{1-\sqrt{3}}$$

$$ك) \frac{\sqrt{1+\sqrt{3}} + \sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}} \cdot \sqrt{\sqrt{3}+\sqrt{2}} \quad (لعل)$$

$$\frac{\sqrt{1+\sqrt{3}} + \sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}} \times \frac{\sqrt{\sqrt{3}+\sqrt{2}}}{\sqrt{\sqrt{3}+\sqrt{2}}} = (\sqrt{1+\sqrt{3}} + \sqrt{\sqrt{3}-1})(\sqrt{\sqrt{3}+\sqrt{2}}) =$$

$$\sqrt{[(\sqrt{1+\sqrt{3}} + \sqrt{\sqrt{3}-1})(\sqrt{\sqrt{3}+\sqrt{2}})]^2} = \sqrt{(1+\sqrt{3} + \sqrt{3}-1)(\sqrt{3}+\sqrt{2})} =$$

$$(\sqrt{3}+\sqrt{2})\sqrt{2} \Rightarrow \sqrt{2}(\sqrt{3}+\sqrt{2}) = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$\sqrt[4]{4^4} \times \sqrt[4]{16^4} \times \sqrt[4]{4^4} = 4^{\frac{4}{4}} \times 4^{\frac{1}{4}} \times 4^{\frac{1}{4}} \times 4^{\frac{1}{4}} \times 4^{\frac{1}{4}} = 4 \times 4 = 16 \quad (ب)$$

$$د) \frac{10^x + 10^{x+1} + 10^{x+2} + 10^{x+3} + 10^{x+4} + 10^{x+5}}{10^{x-2} + 10^{x-1} + 10^x + 10^{x+1} + 10^{x+2} + 10^{x+3}} = 10^2$$

$$\frac{10^x(1+10+10^2+\dots+10^5)}{10^{x-2}(1+10+10^2+\dots+10^5)} = \frac{10^x \left(\frac{10^6-1}{10-1} \right)}{10^{x-2} \left(\frac{10^6-1}{10-1} \right)} = \frac{10^x}{10^{x-2}} = 10^2$$

$$\frac{10^{x+5}}{10^{x-2}} = 10^7 \Rightarrow \frac{10^{x+5}}{10^{x-2}} = 10^7 \Rightarrow x+5 = x-2+7 \Rightarrow x = 2$$

$$4 \quad (a^r + b^r - r_{ab})^r (a^r + b^r + r_{ab})^r = ((a^r + b^r)^r - r_{ab}^r)^r$$

$$(a^r + b^r + r_{ab}^r - r_{ab}^r)^r = (a^r + b^r - r_{ab}^r)^r =$$

$$\left[\frac{(\sqrt[4]{\sqrt{4}-1})^r}{r\sqrt{4}} + \frac{(\sqrt[4]{\sqrt{4}+1})^r}{r\sqrt{4}} - r \frac{(\sqrt[4]{\sqrt{4}-1})(\sqrt[4]{\sqrt{4}+1})}{\sqrt{4}} \right]^r =$$

$$(r\sqrt{4} - r\sqrt{r})^r = [r\sqrt{r}(\sqrt{r}-1)]^r = 14(r-\sqrt{r}) = t(r-\sqrt{r})$$

$$\Rightarrow \boxed{t = 14}$$

$$V \quad a = \sqrt[4]{\sqrt{r}-r\sqrt{r}} = \sqrt{(r-\sqrt{r})^r} = \sqrt{r-\sqrt{r}}$$

$$(a + \frac{1}{a} + \sqrt{r})^r (a + \frac{1}{a} - \sqrt{r})^r = ((a + \frac{1}{a})^r - r)^r =$$

$$(a^r + \frac{1}{a^r} + r - r)^r = (a^r + \frac{1}{a^r})^r = (r - \sqrt{r} + \frac{1}{r - \sqrt{r}})^r =$$

$$(r - \sqrt{r} + r + \sqrt{r})^r = 14 = r^t \Rightarrow \boxed{t = r}$$

$$\Lambda \quad A = \sqrt[4]{r^r \sqrt{14}} \times \left(\frac{1}{r}\right)^{-\frac{r}{2}} = r^{\frac{r}{2}} \times r^{\frac{r}{2}} = r$$

$$(rA)^{\frac{1}{r}} = \Lambda^{-\frac{1}{2}} = \boxed{\frac{1}{r}}$$

9 $\sqrt[p]{V} a^{\frac{1}{\sqrt[p]{V}}} = a^{\frac{1}{\sqrt[p]{V}}} \sim \sqrt[p]{V} \times a^{\frac{1}{\sqrt[p]{V}}} = 1 \sim a = \frac{1}{\sqrt[p]{V}}$

$$\frac{\frac{1}{a} - \sqrt[p]{p}}{1 + \sqrt[p]{p}} = \frac{\sqrt[p]{p} - \sqrt[p]{p}}{1 + \sqrt[p]{p}} \times \frac{\sqrt[p]{p} - 1}{\sqrt[p]{p} - 1} = \frac{p(p - \sqrt[p]{p})}{p} = \boxed{p - \sqrt[p]{p}}$$

10 $(\sqrt{x+a} - \sqrt{x-f}) (\sqrt{x+a} + \sqrt{x-f}) = a+f$

$$\sqrt{x+a} + \sqrt{x-f} = \frac{a+f}{p}$$

$$\sqrt{x+a} + \sqrt{x-f} - p = \frac{a}{p} + p - p = \boxed{\frac{a}{p}}$$