کیمیا مرادی - ۳۱

۱۵،۱۷۵

۱) $1!+3!+5!+\dots+25!$ و $(2!+4!+6!+\dots+26!)$

از $5!$ به بعد رقم یکان ۰ است $5!\ \dots\dots\ 0$

$1!+3!=1+6=7$

$2!+4!=2+2\times3\times4=26$

$6\times7=42 \to 2$ ۲

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۲- الف) $\sqrt[4]{(4+\sqrt7)^{-1}}\ \sqrt{1+\sqrt7}$

$(4+\sqrt7)^{-\frac14}\times(1+\sqrt7)^{\frac12}=$

$A=\frac{1}{(4+\sqrt7)^{\frac14}}\times(1+\sqrt7)^{\frac12}$ | $\left(\frac{(1+\sqrt7)^2}{2}\right)^{\frac14}=\frac{(1+\sqrt7)^{\frac12}}{2^{\frac14}}$

$4+\sqrt7=\frac{(1+\sqrt7)^2}{2}$

$A=\frac{1}{\underbrace{\frac{(1+\sqrt7)^{\frac12}}{2^{\frac14}}}_{a}}\times(1+\sqrt7)^{\frac12}=\frac{2^{\frac14}}{\cancel{(1+\sqrt7)^{\frac12}}}\times\cancel{(1+\sqrt7)^{\frac12}}=A=2^{\frac14}=\sqrt[4]{2}$

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ب) $\overbrace{\left(\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\right)}^{a}\underbrace{(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5})}_{b}$

$b^2=3-\sqrt5+3+\sqrt5-2\sqrt{9-5}$

$b^2=2\to b=\pm\sqrt2$

$\to b=-\sqrt2$ قابل قبول

$\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}=\frac{(\sqrt2+\sqrt5)(\sqrt{10}-2)}{(\sqrt{10}+2)(\sqrt{10}-2)}=$ $\sqrt2\times\sqrt{10}-2\sqrt2+\sqrt5\cdot\sqrt{10}-2\sqrt5$

$10-4=6$

$\frac{\sqrt{20}}{2\sqrt5}-2\sqrt2+\frac{\sqrt{50}}{5\sqrt2}-2\sqrt5$

$(2\sqrt5-2\sqrt5)+(-2\sqrt2+5\sqrt2)$

$=0+3\sqrt2$

$\frac{3\sqrt2}{6}=\frac{\sqrt2}{2}$

$\frac{\sqrt2+\sqrt5}{\sqrt2(\sqrt5+\sqrt2)}=\frac{1}{\sqrt2}$

$\frac{1}{\sqrt2}(-\sqrt2)=-1$

$\frac{\sqrt2}{2}\times(3-\sqrt5)^{\frac12}-(3+\sqrt5)^{\frac12}$

$\sqrt{3\pm\sqrt5}=\frac{(\sqrt{10}\mp\sqrt2)^2}{4}=\frac{10+2\pm2\sqrt{20}}{4}=12\pm4\sqrt5=4(3\pm\sqrt5)$

$\sqrt5+3$ و $\frac{(\sqrt5-1)^2}{2}=\sqrt5-3$

$(\sqrt5-1)^2=5+1-2\sqrt5=6-2\sqrt5=2(3-\sqrt5)$

$3-\sqrt5=\frac{(\sqrt5-1)^2}{2}$

$3+\sqrt5=\frac{(\sqrt5+1)^2}{2}$

جایگذاری $A=\frac{\sqrt2}{2}\cdot\frac{\sqrt5-1}{\sqrt2}-\frac{\sqrt5+1}{\sqrt2}=\frac{\sqrt5-1}{2}-\cancel{\frac{\sqrt5+1}{\sqrt2}}$

$$\frac{\sqrt{16}-\sqrt{4}-2\sqrt{5}-4}{2\sqrt{2}}$$

-3

1,25

1

الف) $\dfrac{2\sqrt{2}+3\sqrt{3}}{5-\sqrt{6}}\times\dfrac{5+\sqrt{6}}{5+\sqrt{6}}=\dfrac{(\sqrt{8}+\sqrt{27})(5+\sqrt{6})}{25-6}=$

$$\frac{2}{\sqrt[4]{9}-1}=\frac{2}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}=\sqrt{3}+1$$

$\sqrt[4]{9}=\sqrt{3}$

$$\frac{(2\sqrt{2}+3\sqrt{3})(5+\sqrt{6})}{19}-\frac{19\sqrt{3}+19}{19}=\sqrt{2}-1$$

ب) $\dfrac{\sqrt{27}-1}{4+\sqrt{3}}\times\dfrac{4-\sqrt{3}}{4-\sqrt{3}}=\dfrac{13\sqrt{3}-13}{13}\to\sqrt{3}-1$

$+\cancel{-12\sqrt{3}-15}$

$$\frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}=-2-\sqrt{3}$$

-4

الف) $B\ \dfrac{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}}{C\ \sqrt{\sqrt{3}-\sqrt{2}}}-2=$

$$\sqrt{\sqrt{3}-1}+\sqrt{1+\sqrt{3}}=x$$

$$x^2=(1+\sqrt{3})+(-1+\sqrt{3})+2\sqrt{(1+\sqrt{3})(\sqrt{3}-1)}=2\sqrt{3}+2\sqrt{\underbrace{(1+\sqrt{3})(-1+\sqrt{3})}_{3-1=2}}$$

$$x^2=2\sqrt{3}+2\sqrt{2}\to 2(\sqrt{3}+\sqrt{2})$$

$$x=\sqrt{2(\sqrt{3}+\sqrt{2})}$$

3

$$\left(\frac{\sqrt{2(\sqrt{3}+\sqrt{2})}}{\sqrt{\sqrt{3}-\sqrt{2}}}\right)-2$$

فقط کسر را حل کنیم

$$\sqrt{\frac{2(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})}\times\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}}=5+2\sqrt{6}$$

$$\frac{2(\sqrt{3}+\sqrt{2})}{\sqrt{3}-\sqrt{2}}=2\times(5+2\sqrt{6})=10+4\sqrt{6}$$

$$\sqrt{10+4\sqrt{6}}-2\qquad\frac{\sqrt{2(\sqrt{3}+\sqrt{2})}}{\sqrt{\sqrt{3}-\sqrt{2}}}=\sqrt{6}+2$$

$$A=\sqrt{6}+2-2=\sqrt{6}$$

ب) $\sqrt[12]{48} \to \sqrt[3]{24}$

$\sqrt[4]{3^4\times 2} \to 3\sqrt[4]{2}$

$\sqrt{4\sqrt[3]{2}} \to \sqrt[12]{2^{13}}$

$2^{\frac{2}{3}}\times 3\times 2^{\frac{1}{4}}\times 2^{\frac{13}{12}} = 2^{2}\times 3 = 12$

۵ـ

$$\frac{3^x(\cancel{3^0+3+3^2+3^3+3^4+3^5})^{364}}{2^x(\cancel{2^{-2}+2^{-1}+2^0+2^1+2^2+2^3})} = 54$$

$\frac{1}{4}+\frac{2}{4}=\frac{3}{4}+1=\frac{7}{4}+\frac{8}{4}=\frac{15}{4}+$

$$\frac{3^x\times 364}{2^x\times 15/4} = 54 \to 3^x\times 364 = 2^x\times 819$$

$\frac{819}{364} = 2{,}25 = \frac{9}{4}$

$\frac{3^x}{2^x}=\frac{9}{4}$

$x=2$

2

۶ـ

$a=\sqrt[4]{\sqrt{6}-2}\quad ,\quad b=\sqrt[4]{\sqrt{6}+2}$

2

$(a^2+b^2-2ab)^2(a^2+b^2+2ab)^2 = \pm(2-\sqrt{3})$

$(\sqrt{(\sqrt{6}-2)}+\sqrt{\sqrt{6}+2}-2\times\sqrt[4]{2})^2(\sqrt{\sqrt{6}-2}+\sqrt{\sqrt{6}+2}+2\times\sqrt[4]{2})^2=$

$(\sqrt{6}+2+\sqrt{6}-2-2\sqrt{2})^2=(2\sqrt{6}-2\sqrt{2})^2=$

$4\,(4+2-4\sqrt{3}) = 8(4-2\sqrt{3}) = 16(2-\sqrt{3})$

گزینه — ۲ $\left((x+\frac{1}{x})^2-2\right)^2 =$ ۱۱۵ −۷

$\left(x^2+\frac{1}{x^2}+2-2\right)^2 = x^4+\frac{1}{x^4}+2$

$7-4\sqrt{3}+\frac{1}{7-4\sqrt{3}}+2 = 9-4\sqrt{3}+\frac{7+4\sqrt{3}}{49-48} =$

$\frac{16}{2}$

برابر ۲ است

$A = \sqrt[5]{2^2\sqrt[3]{2^4}} \times 2^{\frac{4}{15}} - \sqrt[5]{\underbrace{2^2\times 2^{\frac{4}{3}}}_{2^{\frac{10}{3}}}} \times 2^{\frac{4}{3}}$ −۸

$\left(2^{\frac{10}{3}}\right)^{\frac{1}{5}} \times 2^{\frac{4}{15}} - 2^{\frac{2}{3}} \times 2^{\frac{4}{3}}$ ، $2^{\frac{4}{3}}\times 2^{\frac{4}{3}}$

2^2

$(2A)^{-\frac{1}{2}} = 2^{-1} = 0/5$

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−۹

$\sqrt[7]{x} = 27x^{\frac{15}{7}} \xrightarrow{7} x = 27^7 x^{15}$

$\frac{x^{15}}{x} = (3^3)^{-7} \to x^{14} = 3^{-21} \to (x^{14})^{\frac{1}{14}} = (3^{-21})^{\frac{1}{14}}$

$x = 3^{\frac{-21}{14}} = 3^{\frac{-3}{2}} = \frac{1}{3^{\frac{3}{2}}} = \frac{1}{\sqrt{27}} = \frac{1}{3\sqrt{3}}$

$\frac{1}{x} - 3 = 3\sqrt{3} - 3 = 3(\sqrt{3}-1)$

$\frac{3(\sqrt{3}-1)}{\sqrt{3}+1} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)} = \frac{3\times(3+1-2\sqrt{3})}{2} = \frac{3}{2}(4-2\sqrt{3})$

$\boxed{9-3\sqrt{3}} \;\; 3(2-\sqrt{3})$

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15-

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$$\underbrace{\sqrt{x+a}}_{A} - \underbrace{\sqrt{x-4}}_{B} = 2$$

$$A - B = 2$$

$$(a+b) - 2 = \frac{a+4}{2} - \frac{4}{2} = \frac{a}{2}$$

$$\underbrace{(a-b)}_{2}(a+b) = a^2 - b^2 \Rightarrow 2(a+b) = a^2 - b^2 = 2(a+b)$$

$$a+4$$

$$2(a+b) = a+4 \Rightarrow a+b = \boxed{\frac{a+4}{2}}$$

$$(\sqrt{x+a}+\sqrt{x-4})(\sqrt{x+a}-\sqrt{x-4}) = (x+a)-(x-4)$$

$$a+4$$

$$(\sqrt{x+a}+\sqrt{x-4}) = \frac{a}{2}+2$$

$$\sqrt{x+4} + \sqrt{x-4} - 2 = \frac{a}{9} + 2 - 2 = \frac{a}{2}$$