کیمیا مرادی - ۳۱

۱)

د) $(1!+3!+5!+\dots+25!)(2!+4!+6!+\dots+24!)$

از ۵! به بعد رقم یکان ۰ است $5!\ \dots\dots$

$1!+3!=1+6=7$

$2!+4!=2+2\times3\times4=26$

$26\times7=182\rightarrow 2$

۲-

الف) $\sqrt[4]{(4+\sqrt7)^{-1}}\ \sqrt{1+\sqrt7}$

$(4+\sqrt7)^{-\frac14}\times(1+\sqrt7)^{\frac12}=$

$A=\frac{1}{(4+\sqrt7)^{\frac14}}\times(1+\sqrt7)^{\frac12}$ | $\left(\frac{(1+\sqrt7)^2}{2}\right)^{\frac14}=\frac{(1+\sqrt7)^{\frac12}}{2^{\frac14}}$

$4+\sqrt7=\frac{(1+\sqrt7)^2}{2}$

$A=\frac{1}{\frac{(1+\sqrt7)^{\frac12}}{2^{\frac14}}}\times(1+\sqrt7)^{\frac12}=\frac{2^{\frac14}}{\cancel{(1+\sqrt7)^{\frac12}}}\times\cancel{(1+\sqrt7)^{\frac12}}=A=2^{\frac14}=\sqrt[4]{2}$

ب) $\left(\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\right)(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5})$

$\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}=\frac{(\sqrt2+\sqrt5)(\sqrt{10}-2)}{(\sqrt{10}+2)(\sqrt{10}-2)}=$ ۶ $\sqrt2\times\sqrt{10}-2\sqrt2+\sqrt5\cdot\sqrt{10}-2\sqrt5$

$10-4=6$

$\frac{\cancel{\sqrt{20}}}{2\sqrt5}-2\sqrt2+\frac{\cancel{\sqrt{50}}}{5\sqrt2}-2\sqrt5$

$\frac{3\sqrt2}{6}=\frac{\sqrt2}{2}$

$(2\sqrt5-2\sqrt5)+(-2\sqrt2+5\sqrt2)$

$=0+3\sqrt2$

$\frac{\sqrt2}{2}\times(3-\sqrt5)^{\frac12}-(3+\sqrt5)^{\frac12}$

$\sqrt{3\pm\sqrt5}=\frac{(\sqrt{10}\mp\sqrt2)^2}{4}=\frac{10+2\pm2\sqrt{20}=12\pm4\sqrt5=4(3\pm\sqrt5)}{}$

$\sqrt5+3$ و $\frac{(\sqrt5-1)^2}{2}=\sqrt5-3$

$(\sqrt5-1)^2=5+1-2\sqrt5=6-2\sqrt5=2(3-\sqrt5)$

$3-\sqrt5=\frac{(\sqrt5-1)^2}{2}$

$3+\sqrt5=\frac{(\sqrt5+1)^2}{2}$

جایگذاری $A=\frac{\sqrt2}{2}\cdot\frac{\sqrt5-1}{\sqrt2}-\frac{\sqrt5+1}{\sqrt2}=\frac{\sqrt5-1}{2}-\frac{\sqrt5+1}{\sqrt2}$

$$\frac{\sqrt{15}-\sqrt{6}-2\sqrt{5}-2}{2\sqrt{3}}$$

۳- الف) $\dfrac{2\sqrt{2}+3\sqrt{3}}{5-\sqrt{6}} \times \dfrac{5+\sqrt{6}}{5+\sqrt{6}} = \dfrac{\cancel{(\sqrt{8}+\sqrt{27})}(5+\sqrt{6})}{25-6}$ $\quad 2\sqrt{2}+3\sqrt{3}$ $=$

$$\frac{2}{\cancel{\sqrt[4]{9}}-1} = \frac{2}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1} = \sqrt{3}+1$$

$\sqrt[4]{9} = \sqrt{3}$

$$\frac{(2\sqrt{2}+3\sqrt{3})(5+\sqrt{6})}{19} - \frac{19\sqrt{3}+19}{19} = \sqrt{2}-1$$

ب) $\dfrac{\sqrt{27}-1}{4+\sqrt{3}} \times \dfrac{4-\sqrt{3}}{4-\sqrt{3}} = \dfrac{13\sqrt{3}-13}{13}$

$\dfrac{1}{2-\sqrt{3}} \times \dfrac{2+\sqrt{3}}{2+\sqrt{3}} = -2-\sqrt{3}$

$+ \longrightarrow 12\sqrt{3}-15$

۴- الف) $B = \dfrac{\sqrt{1+\sqrt{3}}+\sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}} - 2 =$

C

$$\sqrt{\sqrt{3}-1} + \sqrt{1+\sqrt{3}} = x$$

$$x^2 = (1+\sqrt{3}) + (-1+\sqrt{3}) + 2\sqrt{(1+\sqrt{3})(\sqrt{3}-1)} = 2\sqrt{3} + 2\underbrace{\sqrt{(1+\sqrt{3})(1-\sqrt{3})}}_{3-1=2}$$

$$x^2 = 2\sqrt{3}+2\sqrt{2} \rightarrow 2(\sqrt{3}+\sqrt{2})$$

$$x = \sqrt{2(\sqrt{3}+\sqrt{2})}$$

$$\left(\frac{\sqrt{2(\sqrt{3}+\sqrt{2})}}{\sqrt{\sqrt{3}-\sqrt{2}}}\right) - 2$$

[illegible] $\longrightarrow \sqrt{\dfrac{2(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})} \times \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}} = 5+2\sqrt{6}$

$$\frac{2(\sqrt{3}+\sqrt{2})}{\sqrt{3}-\sqrt{2}} = 2\times(5+2\sqrt{6}) = 10+4\sqrt{6}$$

$$\sqrt{10+4\sqrt{6}} - 2 \quad \frac{\sqrt{2(\sqrt{3}+\sqrt{2})}}{\sqrt{\sqrt{3}-\sqrt{2}}} = \sqrt{6}+2$$

$$A = \sqrt{6}+2-2 = \sqrt{6}$$

ب) $\sqrt[12]{28} \to \sqrt[3]{24}$

$\sqrt[4]{3^4 \times 2} \to 3\sqrt[4]{2}$

$\sqrt{4\sqrt[6]{2}} \to \sqrt[12]{2^{13}}$

$2^{\frac{2}{3}} \times 3 \times 2^{\frac{1}{4}} \times 2^{\frac{13}{12}} = 2^{2} \times 3 = \underline{12}$

۵-

$$\frac{3^x(\overset{364}{\cancel{3^0+3+3^2+3^3+3^4+3^5}})}{2^x(\cancel{2^{-2}+2^{-1}+2^0+2^1+2^2+2^3})} = 54$$

$\frac{1}{4}+\frac{2}{4}=\frac{3}{4}+1=\frac{7}{4}+\frac{8}{4}=\frac{15}{4}+$

$$\frac{3^x \times 364}{2^x \times 15/75} = 54 \to 3^x \times 364 = 2^x \times 819$$

$\frac{819}{364} = 2/25 = \frac{9}{4}$

$\frac{3^x}{2^x} = \frac{9}{4}$

$x = 2$

۶-

$a = \sqrt[4]{\sqrt{6}-2}\ ,\ b = \sqrt[4]{\sqrt{6}+2}$

$(a^2+b^2-2ab)^2(a^2+b^2+2ab)^2 = \pm(2-\sqrt{3})$

$(\sqrt{(\sqrt{6}-2)}+\sqrt{\sqrt{6}+\sqrt{2}}-2\times\sqrt[4]{2})^2(\sqrt{\sqrt{6}-2)}+\sqrt{\sqrt{6}+\sqrt{2}}+2\times\sqrt[4]{2})^2 =$

$(\sqrt{6}+2+\sqrt{6}-2-2\sqrt{2})^2 = (2\sqrt{6}-2\sqrt{2})^2 =$

$$4(9+2-4\sqrt{3})=8(4-2\sqrt{3})=\underline{16(2-\sqrt{3})}$$

گزینه -2 $\qquad$ -7

$$\left(\left(x+\frac{1}{x}\right)^2-2\right)^2=$$

$$\left(x^2+\frac{1}{x^2}+2-2\right)^2=x^4+\frac{1}{x^4}+2$$

$$7-4\sqrt{3}+\frac{1}{7-4\sqrt{3}}+2=9-4\sqrt{3}+\frac{7+4\sqrt{3}}{49-48}=$$

$$\frac{16}{7}$$

-8

$$A=\sqrt[5]{2^2\sqrt[3]{2^4}}\times 2^{\frac{4}{15}}-3\sqrt[5]{\underbrace{2^2\times 2^{\frac{4}{3}}}_{2^{\frac{10}{3}}}}\times 2^{\frac{4}{3}}$$

$$\left(2^{\frac{10}{3}}\right)^{\frac{1}{5}}\times 2^{\frac{4}{3}}-6\,2^{\frac{2}{3}}\times 2^{\frac{4}{15}}$$

$$2^2$$

$$(2A)^{\frac{-1}{2}}=2^{-1}=0/5$$

-9

$$\sqrt[7]{x}=27x^{\frac{15}{7}}\xrightarrow{7}x=27^7x^{15}$$

$$\frac{x^{15}}{x}=(3^3)^{-7}\longrightarrow x^{14}=3^{-21}\longrightarrow(x^{14})^{\frac{1}{14}}=(3^{-21})^{\frac{1}{14}}$$

$$x=3^{\frac{-21}{14}}=3^{\frac{-3}{2}}=\frac{1}{3^{\frac{3}{2}}}=\frac{1}{\sqrt{27}}=\frac{1}{3\sqrt{3}}$$

$$\frac{1}{x}-3=3\sqrt{3}-3=3(\sqrt{3}-1)$$

$$\frac{3(\sqrt{3}-1)}{\sqrt{3}+1}\times\frac{(\sqrt{3}-1)}{(\sqrt{3}-1)}=\frac{3\times(3+1-2\sqrt{3})}{2}=\frac{3}{2}(4-2\sqrt{3})$$

$$\boxed{6-3\sqrt{3}}\leftarrow 3(2-\sqrt{3})$$

$$\underbrace{\sqrt{x+a}}_{A} - \underbrace{\sqrt{x-۴}}_{B} = ۲$$

$$A-B=۲$$

$$(a+b)-۲ = \frac{a+۴}{۲} - \frac{۴}{۲} = \frac{a}{۲}$$

$$\underbrace{(a-b)}_{۲}(a+b) = a^۲ - b^۲ \rightarrow ۲(a+b) = a^۲ - b^۲ = ۲(a+b)$$

$$a+۴$$

$$۲(a+b) = a+۴ \Rightarrow a+b = \boxed{\frac{a+۴}{۲}}$$