19

(۱)

$x^2-5x+2=0$

$\alpha\beta=2 \Rightarrow \alpha=\frac{2}{\beta}$

$\frac{2\alpha+\beta^3}{5\beta^2} \Rightarrow \frac{2\alpha}{5\beta^2}+\frac{\beta^3}{5\beta^2} \Rightarrow \frac{2\alpha}{5\beta^2}+\frac{\beta^{10}\times\beta^2}{5\times\beta^{10}}=$

$\beta^2-5\beta+2=0 \Rightarrow \beta^2=5\beta-2$

$\beta^{12}\Rightarrow\beta\beta^2=\beta(5\beta-2)=5\beta^2-2\beta$

$5(5\beta-2)-2\beta=23\beta-10$

$\beta^5=\beta^{12}\beta^2=(23\beta-10)(5\beta-2)=87\beta-30$

$\times\beta\left(\frac{2(\frac{2}{\beta})+87\beta-230}{5(5\beta-2)}\right)=$

(۲) ✓

$\frac{8+87\beta^2-230\beta}{25\beta^2-10\beta}=\frac{87(5\beta-2)-230\beta+8}{25(5\beta-2)-10\beta}=\frac{235\beta-90}{115\beta-50}\overset{\div 5}{\Rightarrow}\frac{47\beta-18}{23\beta-10}=\frac{19(23\beta-10)}{23\beta-10}=19$

19 ✓

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(۳) در یک سهمی دو نقطه هم عرض داریم. طول میانگین طول دو نقطه است. $x_S=\frac{4}{2}=2$ $\Leftarrow$ $y_S=\frac{3}{2}$

(۲) ✓

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(۴) A, B = متقارن‌اند $\Rightarrow x=\frac{3+(-5)}{2}=-1 \to (-1,1)$ — تقارن نسبت به آن محاسبه شده است

$y=a(x+1)^2+1$

برخورد با محور x $\to y=0$ $\quad a(x+1)^2+1=0 \to (x+1)^2=-\frac{1}{a}$

(۲) ✓

$\alpha^2+\beta^2=2(1+\alpha\beta)-5 \to (1+\alpha)^2=\frac{5}{2}$

$\alpha^2+2\alpha+1=\frac{3}{2}$

$-\frac{2}{3}=a \to a=-\frac{3}{2}$

$y=-\frac{3}{2}(x+1)^2+1$

برخورد با محور y: $x=0 \to y=\frac{1}{2}(1)+1=\frac{3}{2} \to \left(0,\frac{3}{2}\right)$

---

(۵) $a=-1$

$\Delta=25+4m>0 \to m>-\frac{25}{4} \to m>-6/25$

$\frac{5}{2}<$ طول رأس

$-\left(\frac{5}{2}\right)^2+5\left(\frac{5}{2}\right)+m=-\frac{25}{4}+\frac{25}{2}+m=\frac{25}{4}+m$

$\frac{5}{4}+m<0 \Rightarrow m<-\frac{5}{4}$

$m<-3$

(۲) ✓

(۴) صحیح نیست $\Leftarrow$ $m=-6-5-4-2$

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(۶) $y=ax^2+bx+c$ $\quad a=m$ $\quad x_0=-\frac{b}{2a}$ $\quad b=-12$ $\quad x=-\frac{-12}{2m}=\frac{12}{2m}=\frac{6}{m}$

$y_{min}=m\left(\frac{6}{m}\right)^2-12\left(\frac{6}{m}\right)+5m-1$

$\to y_{min}=\frac{36}{m}-\frac{72}{m}+5m-1$

$-\frac{36}{m}+5m-1=2 \Rightarrow -\frac{36}{m}+5m=3$ $\quad \times m$

$5m^2-36=3m$

$5m^2-3m-36=0$

✓

$(5m+12)(m-3)=0$

$m=-\frac{12}{5}$ $\quad$ $\boxed{m=3}$

کمترین مقدار صحیح رو به بالا است

$\frac{6}{m} \Rightarrow \frac{6}{3}=\textcircled{2}$

۲

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(۳)

$|\alpha-\beta| = \frac{\sqrt{b^2-4ac}}{|a|}$ $\quad a=1,\ b=2k,\ c=5$

$\Delta = 4k^2-20 \Rightarrow |\alpha-\beta| = \sqrt{4k^2-20}$

$\sqrt{4k^2-20} = \frac{4}{3}k \Rightarrow \left(4k^2-20 = \frac{16}{9}k^2\right)$ ضرب ۹× $= 36k^2-180 = 16k^2$

$20k^2 = 180$

$k^2 = 9$

$\frac{k^2}{2} = \frac{9}{2} = [4.5] = \boxed{4}$

۲ ✓

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(۷)

$a, \alpha, \beta \to a, aq, aq^2$

$a^2 = \alpha \cdot \beta$

$\alpha+\beta = -2(a+1)$ $\quad aq+aq^2 = aq(1+q) = -2(a+1)$ $\quad$ (1)

$\alpha\beta = 2a-1$ $\quad aq \times aq^2 = a^2q^3 = 2a-1$ $\quad q^3 = \frac{2a-1}{a^2}$ امتحان کردن $\longrightarrow \boxed{a=-\frac{1}{2}}$ $\quad q^3=-8 \to q=-2$

$a^2 = 2a-1 \to (a-1)^2 = 0 \to a=1$

$aq(1+q) = -1 = -2(a+1)$

$a=1 \to x^2+4x+1=0$ $\quad \Delta<0$

---

(۸)

$x^4-7x^2-5=0$ $\quad t^2-7t-5=0$ $\quad t=\frac{7\pm\sqrt{69}}{2} \Rightarrow t_1=\frac{7+\sqrt{69}}{2}$ ✓ $\quad t_2=\frac{7-\sqrt{69}}{2}$ ✗

$x^2=t$ $\quad \Delta=49+20=69$

$x^2=\frac{7+\sqrt{69}}{2}$ $\quad x_{1,2}=\pm\sqrt{\frac{7\pm\sqrt{69}}{2}}$ $\quad S=0$

$P=\frac{7+\sqrt{69}}{2}$

$2P^2-3SP+4S = 2P^2 = 2\left(\frac{49+69+14\sqrt{69}}{4}\right) = \frac{118+14\sqrt{69}}{2} = \boxed{59+7\sqrt{69}}$

۲ ✓

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(۹)

$x_{رأس} = \frac{-b}{2a} = -\frac{-4}{2k} = +\frac{2}{k}$

$y = -4x-4$ جایگذاری $\frac{4-6k}{k} = -4\left(\frac{2}{k}\right)-4 \xrightarrow{\text{ضرب در } k} 4-6k = -8-4k$

$y_{رأس} = \frac{-\Delta}{4a} = \frac{16-24k}{4k} = -\frac{-4+6k}{k}$

$-2k = -4$

$\boxed{k=2}$

$y_{رأس} = -\frac{-4+6\times 2}{2} = -\frac{16}{2} = \boxed{-8}$

۲ ✓

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(۱۰)

$-mx^2+mx+1 = -m-x$

هیچ نقطه‌ای دیگر را قطع نمی‌کنند $\Rightarrow \Delta<0$

$-mx^2+mx+x+m+1=0$

$\times(-1)$: $mx^2-mx-x-m-1=0$

$-(m+1)x \quad -(m+1)$

$\Delta = (m+1)^2+4m(m+1) = (m+1)(5m+1)$

$(m+1)(5m+1)<0 \longrightarrow -1<m<-\frac{1}{5}$

$5m^2+6m+1$

$(m+1)(m+\frac{1}{5})$

$-\frac{1}{5} \quad -\frac{5}{5}$

هیچ عدد صحیح در این بازه نیست

۲ ✓

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