هلیارب

۱۹ تا

۱) $\alpha = 5 - \beta \quad \alpha + \beta = 5 \quad \alpha\beta = 2$

$\beta^2 = 5\beta - 2 \to \beta^4 = 5\beta^3 - 2\beta \to \beta^5 = 5\beta^4 - 2\beta^2 = 24\beta - 10$

$\alpha^5 + \beta^5 = 4(5 - \beta) + 24\beta - 10 = 10 + 20\beta$

$5\beta = \beta^2 + 2 \Rightarrow 20\beta = 4\beta^2 + 8$

$\to 10 + 20\beta = 4\beta^2 + 18$

$45\beta^2 + 18 \to -10 + 20\beta = 45\beta^2$

$\alpha^5 + \beta^5$

$\frac{45\beta^2}{5\beta^2} = 9$ ✓

۲) $\frac{4 - 1.5}{2} = 1.25 \Rightarrow -\frac{b}{2a}$ ، عرض رأس $= -\frac{b}{a} = 1.5$ ✓

۳) $(\alpha - \beta) = \frac{\sqrt{5}}{|a|} \Rightarrow \frac{4}{3} K = \frac{\sqrt{4(k^2 - 5)}}{1} = 2\sqrt{k^2 - 5} \Rightarrow \frac{2}{3} K = \sqrt{k^2 - 5}$

توان ۲ $\to \frac{4}{9} k^2 = k^2 - 5 \Rightarrow a = \frac{5}{9} k^2 \Rightarrow k^2 = 9 \Rightarrow \left[\frac{k^2}{2}\right] = \left[\frac{9}{2}\right] = 4$ ✓

۴) $A \begin{vmatrix} 2 \\ y \end{vmatrix}, B \begin{vmatrix} -1 \\ 1 \end{vmatrix} \quad \frac{ab}{2} = \frac{-a+2}{2} = -1$

$a^2 + b^2 = 5 \Rightarrow (a+b)^2 - 2ab = 5 \Rightarrow ab = -\frac{1}{2}$

$y = a(x^2 + 2ax - \frac{1}{2}) = 1 \Rightarrow a(1 - 2 - \frac{1}{2}) \Rightarrow a = -\frac{2}{3}$

$y = \frac{1}{3}$ ✓

۵) $\alpha + \beta$ کوتاهترین $= \alpha + \beta$ $\quad \frac{\alpha + \beta}{2} = 2.5$

$\Delta \geq 0 \Rightarrow 24 + 4m \geq 0 \quad 4m \geq -24$

$m \geq -6$

$\frac{-5 + \sqrt{24 + 4m}}{-2} > \frac{9}{2}$

$\sqrt{24 + 4m} > -4 \to$ همواره برقرار

$\frac{-5 - \sqrt{24 + 4m}}{-2} > 0 \quad \frac{9}{2}$

$(\sqrt{24 + 4m})^2 < (4)^2$

$24 + 4m < 16$

$4m < -8$

$m < -2$

$-6 < m < -2$

$[-6, -5, -4, -3]$ ✓

۶) $y_{min} = \frac{-\Delta}{4a} = -2 \Rightarrow 2 = \frac{20m^2 - 4(m)(m) - 144}{4m} \Rightarrow 2 \to \frac{4(5m^2 - m - 36)}{4m}$

$\frac{2 = 5m^2 - m - 36}{m} \Rightarrow 2 = 5m^2 - m - 36 \Rightarrow 5m^2 - m - 38 = 0$

$5m^2 - m - 38 = 0 \to m = \frac{1 \pm \sqrt{1 - 4(5)(-38)}}{10} \Rightarrow \frac{1 \pm \sqrt{761}}{10} = \frac{1 \pm 27}{10}$

$\begin{cases} m = 2 \checkmark \\ m = \frac{-13}{5} \times \end{cases}$ طول مینیمم $= \frac{-b}{2a} = \frac{12}{4m} = 2$ ✓

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۷) $\alpha + \beta = -2a - 2$ $\alpha \cdot \beta = 2a - 1$

$a^2 = \alpha \cdot \beta = 2a - 1$ $a^2 - 2a + 1 = 0 \to (a-1)^2 = 0 \to a = 1$

شرط وجود دو ریشه برای $a=1$ بررسی نشد

$a=1 \to x^2 + 4x + 1 = 0 \to \Delta > 0$ ✓

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۸) $t^2 - 7t - 5 = 0$

$\Delta = 49 - 4(-5) = 69$

$t_1 = \frac{7 - \sqrt{69}}{2}$ یا $t_2 = \frac{7 + \sqrt{69}}{2} \Rightarrow$ ✓

$x_1, x_2 = \pm \sqrt{\frac{7 + \sqrt{69}}{2}}$

$d = \frac{7 + \sqrt{69}}{2}$

$S^2 = 4(\frac{69 + 14\sqrt{69} + 121}{4}) = \frac{190 + 14\sqrt{69}}{2}$

$S = \sqrt{95 + 7\sqrt{69}}$

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۹) $ext \begin{cases} x = \frac{-3}{14} = \frac{-2}{14} \\ y = 14(\frac{2}{14})^2 - 6(\frac{2}{14}) = \frac{-3}{14} \end{cases}$

$\frac{-3}{14} - 6 = -4(\frac{1}{7}) \Rightarrow \frac{3}{14} - 2 \Rightarrow k < 14$

$y = \frac{-3}{14} - 6 \Rightarrow -7$

$\begin{vmatrix} \frac{-2}{14} \\ \frac{3}{14} \\ \frac{-3}{14} - 6 \end{vmatrix}$ $y = -3m - 3$

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۱۰) $-mx^2 + 2mx - 1 = -x - m \Rightarrow -mx^2 + (2m+1)x + m - 1 = 0$

$\Delta < 0 \Rightarrow b^2 - 4ac < 0 \Rightarrow (2m-1)^2 - 4(-m)(m-1) < 0$

$4m^2 + 1 - 4m + 4m^2 - 4m < 0$

$8m^2 - 8m + 1 < 0$

$a + c = b$

$x = -1 \quad , \quad x = \frac{-c}{a} = \frac{-1}{8} = -0.125$

$-1 < m < -0.125$

	$-\infty$		-1		-0.125		$+\infty$
	+		−		+		

✓