★ آنالیز جبری - دهم تجربی A - تالیف ۲۲ آبان

$\mapsto \alpha = 5 - \beta$

$\alpha + \beta = 5,\ \alpha\beta = 2$ (۱)

$\beta^2 = 5\beta - 2 \rightarrow \beta^3 = 5\beta^2 - 2\beta = 5(5\beta - 2) - 2\beta = 23\beta - 10$

$\beta^4 = 23\beta^2 - 10\beta = 23(5\beta - 2) - 10\beta = 105\beta - 46 \rightarrow \beta^5 = 105\beta^2 - 46\beta = 105(\beta - 2) - 46\beta$

$\Rightarrow \beta^5 = 479\beta - 210$

$$\frac{4\alpha + \beta^5}{5\beta^2} = \frac{4\alpha + 479\beta - 210}{5(5\beta - 2)} \overset{\alpha = 5 - \beta}{=} \frac{20 - 4\beta + 479\beta - 210}{25\beta - 10} = \frac{475\beta - 190}{25\beta - 10} = \frac{95(5\beta - 2)}{5(5\beta - 2)} = 19$$ (۱)

$x = \frac{x_1 + x_2}{2} = \frac{3 - 1,5}{2} = 0,75$ = محور تقارن $= \frac{-b}{2a}$ (۲)

$\rightarrow$ شیب خط $= \frac{-b}{a} = ?$ $\rightarrow \frac{-b}{2a} = 0,75 \Rightarrow \frac{-b}{a} = 1,5$

$$|\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} \rightarrow \frac{2}{3}k = \frac{\sqrt{b^2 - 4ac}}{|1|} = \frac{\sqrt{4(k^2 - 5)}}{1} = 2\sqrt{k^2 - 5} \Rightarrow \frac{2}{3}k = \sqrt{k^2 - 5}$$ (۳)

$\xrightarrow{\text{توان ۲}} \frac{4}{9}k^2 = k^2 - 5 \rightarrow 5 = \frac{5}{9}k^2 \rightarrow k^2 = 9 \rightarrow \left[\frac{k^2}{2}\right] = \left[\frac{9}{2}\right] = 4$ (۴)

(۴) $y = ax^2 + bx + c \rightarrow ext \left| \begin{matrix} \frac{-b}{2a} = -1 \\ 1 \end{matrix} \right.$

$\downarrow$

$\begin{cases} y = 25a - 5b + c \\ y = 9a + 3b + c \end{cases}$

$\rightarrow 16a - 8b = 0 \rightarrow 2a = b$

$\alpha^2 + \beta^2 = S^2 - 2P = 5$

$\rightarrow S = \alpha + \beta = \frac{-b}{a} = -2 \Rightarrow S^2 = 4$

$\Rightarrow \frac{S^2}{4} - 2P = 5 \Rightarrow P = -\frac{1}{2} \Rightarrow y = x^2 - Sx + P$

$\Downarrow$

$y = x^2 + 2x - \frac{1}{2}$

$\rightarrow y = a(x^2 + 2x - \frac{1}{2}) \rightarrow 1 = a(1 - 2 - \frac{1}{2}) \rightarrow a = -2 \rightarrow y = -2(x^2 + 2x - \frac{1}{2})$

$\Rightarrow y = 1 - 2x^2 - 4x$

Scanned with CamScanner

Subject: Date: Sa Su Mo Tu We Th Fr

۵- $\alpha+\beta=5 \rightarrow$ طول رأس $= \frac{\alpha+\beta}{2} = 2.5$

۱) $\Delta \geq 0 \rightarrow (-5)^2 - 4(1)(-m) \geq 0 \rightarrow 4m \geq -25 \rightarrow m \geq -6.25$

۲) $f(\frac{5}{2}) \geq 0 \rightarrow (2.5)^2 - 5(2.5) - m \geq 0 \rightarrow m \leq -2.25$

$\Rightarrow -6.25 \leq m \leq -2.25 \rightarrow m \in \{-6, -5, -4, -3\}$

۶- $-\frac{\Delta}{4a} = \frac{144 - 4(m)(5m-1)}{4m}$ ، $\frac{m(5m-1)-36}{m} = 2 \rightarrow 5m^2 - m - 36 = 2m$

$\Rightarrow 5m^2 - 3m - 36 = 0 \rightarrow (m-3)(5m+12) = 0 \rightarrow m = 3, \frac{-12}{5}$ ✗

چون تابع min دارد است و بنابراین a باید مثبت باشد.

$y = 3x^2 - 12x + 14$

$\Rightarrow$ طول رأس $= x = \frac{-b}{2a} = \frac{+12}{6} = +2$ ✓

۷- $\alpha+\beta = -2a-2$ ، $\alpha\beta = 2a-1$

$a^2 = \alpha\beta = 2a-1$

اگر α و β و a یک تصاعد هندسی باشند، پس:

$\Rightarrow a^2 - 2a + 1 = 0 \rightarrow (a-1)^2 = 0 \Rightarrow a = 1$ ✓

۸- $t = x^2 \rightarrow t^2 - 7t - 5 = 0 \rightarrow \Delta > 0 \rightarrow 49 - 4(-5) = 69$

$\rightarrow t_1, t_2 = \frac{-b \pm \sqrt{69}}{2a} = \frac{7 \pm \sqrt{69}}{2} \Rightarrow x_1, x_2 = \sqrt{\frac{7+\sqrt{69}}{2}}, -\sqrt{\frac{7+\sqrt{69}}{2}}$

$\Rightarrow S = x_1 + x_2 = 0$

$\Rightarrow P = x_1 \cdot x_2 = -\left(\frac{7+\sqrt{69}}{2}\right)$

$\star \Rightarrow 2P^2 - 4SP + 2S = 2P^2 = 2\left(-\left(\frac{7+\sqrt{69}}{2}\right)\right)^2$

$= 69 + 7\sqrt{69}$ ✓

Scanned with CamScanner

۹- این نقطه روی خط $y=-۴x-۴$ قرار دارد.

$ext \Big| x=\frac{-۴}{۲k}=\frac{-۲}{k}$

$y=k\left(\frac{۲}{k}\right)^۲-۴\left(\frac{۲}{k}\right)-۹=\frac{-۴}{k}-۹$

$\Rightarrow \Big| \frac{-۲}{k} \quad \frac{-۴}{k}-۹$

$\Rightarrow \frac{-۴}{k}-۹=-۴\left(\frac{۲}{k}\right)-۴ \rightarrow \frac{۴}{k}=۲ \rightarrow k=۲$

$\Rightarrow y=\frac{-۴}{k}-۹=-۸$

۱۰-

$-mx^۲+mx+۱=-x-m \rightarrow -mx^۲+(m+۱)x+۱+m=۰$

$\rightarrow \Delta<۰ \rightarrow b^۲-۴ac<۰ \rightarrow (m+۱)^۲-۴(-m)(m+۱)<۰$

$m^۲+۱+۲m+۴m^۲+۴m=۵m^۲+۶m+۱=(۵m+۱)(m$

$\Rightarrow (۵m+۱)(m+۱)<۰ \rightarrow \boxed{-۱<m<-۰,۲} \rightarrow$ هیچ عدد صحیحی نداریم در این بازه.

Scanned with CamScanner