پاسخنامه تکلیف شماره ۲۳ — هلنا عامری — دهم دفتر A

۱) $y=a(x+1)^2+9 \xrightarrow{(3,1)} 1=a(\overset{4}{3+1})^2+9 \to 1=16a+9 \Rightarrow a=\frac{-1}{2}$

$y=\frac{1}{2}(x+1)^2+9 \Rightarrow y=\frac{-1}{2}x^2-x+\frac{17}{2}$

۲) $\Delta>0 \quad S>0 \to \frac{-b}{a}>0 \Rightarrow \frac{-m}{2}>0 \qquad P>0 \Rightarrow \frac{c}{a}>0 \to \frac{c}{a}=\frac{m+6}{2} \to -6$

$\Delta:\ b^2-4ac>0 \to m^2-4(2)(m+6)=m^2-8m-48>0 \qquad (\underbrace{m-12}_{12})(\underbrace{m+4}_{-4})>0$

	-4		12	
+		−		+

$m<0$

$m>-6$

$m<-4,\ m>12$

$\Bigg\}\ \cap \to -6<m<4 \to (-6,4)$

۳) $\frac{-b}{a}=\frac{a}{c} \qquad \frac{-2m+1}{3}=\frac{3}{2-m} \qquad (-2m+1)(2-m)=9 \to -4m+2m^2+2-m=2m^2-5m-7=0$

$\frac{-b\pm\sqrt{\Delta}}{2a}=\frac{5\pm\sqrt{25+56}}{4}=\frac{5\pm 9}{4} \to m_1=\frac{7}{2} \qquad m_2=-1$

$m_2=-1 \to \Delta=(2(-1)-1)^2-4(3)(2-1(-1))=9-36=-27$ ✗

$m_1=\frac{7}{2} \to \Delta=(2(\frac{7}{2})-1)^2-4((3)(2-\frac{7}{2}))=36+18=54$ ✓

۴) $x^2-x-4=0 \Rightarrow S=1,\ P=-4$

$S'=x_1^3+x_2^3+\frac{1}{x_1}+\frac{1}{x_2}=(S^3-3SP)+\frac{\overbrace{x_1+x_2}^{S}}{\underbrace{x_1\times x_2}_{P}}$

$\Rightarrow S'=(1-3\times1\times-4)+\frac{1}{-4}=13-\frac{1}{4}=\frac{51}{4}$

$P'=(x_1^3+\frac{1}{x_2})(x_2^3+\frac{1}{x_1})=x_1^3x_2^3+\frac{1}{x_1x_2}+x_1^2+x_2^2=-64-\frac{1}{4}+9=-\frac{221}{4}$

معادله جدید $\to x^2-Sx+P=0 \qquad x^2-\frac{51}{4}x-\frac{221}{4}=0 \to \boxed{4x^2-51x-221=0}$

$\sqrt[3]{x^2}=t \qquad (t^2+\frac{1}{t^2}+1)(t^2-1)=2t \to (\frac{t^4+t^2+1}{t^2})(t^2-1)=2t$ ۵_

$\frac{(t^2)^3-1}{t^2}=2t \to \frac{t^6-1}{t^2}=2t \to t^6-1=2t^3 \qquad t^6-2t^3-1=0 \qquad t^3=z \to z^2-2z-1=0$

$z=\frac{2\pm\sqrt{8}}{2}=\boxed{1\pm\sqrt{2}=t^3} \to t=\sqrt[3]{x} \Rightarrow t^3=x \Rightarrow \boxed{x=1\pm\sqrt{2}}$

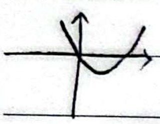
$\Rightarrow x_1+x_2=1+\sqrt{2}+1-\sqrt{2}=\boxed{2}$

$x_1+x_2=\frac{a}{3}$ ① $\quad x_2=3x_1 \qquad x_1x_2=\frac{4}{3}$ ② $\qquad x_1(3x_1)=\frac{4}{3} \to x_1^2=\frac{4}{9}$ ۶_

$\boxed{x=\pm\frac{2}{3}}$

$\underbrace{x_1+3x_1=\frac{a}{3}}$

$4x_1=\frac{a}{3} \to a=12x_1 \qquad \begin{cases} a_1=12(\frac{2}{3})=8 \;,\; a_2=12(-\frac{2}{3})=-8 \\ \Rightarrow |a_1-a_2|=|8-(-8)|=\boxed{16} \end{cases}$



$\boxed{a>0} \qquad \underbrace{S>0} \qquad P=0 \qquad \underline{\Delta>0}$ ۷_

$-\frac{b}{a}>0 \Rightarrow \frac{-3-2a}{a}>0 \qquad (3+2a)^2>0 \to$ همواره برقرار

$\to \begin{array}{c} -\frac{3}{2} \quad 0 \\ \hline - \quad + \quad - \end{array} \qquad (-\frac{3}{2},0)\cap(0,+\infty)=\varnothing$ به ازای هیچ مقدار

$\frac{-b}{2a}=\frac{-a}{2}=\frac{2}{-2}=-1 \Rightarrow \boxed{a=2}$ سهمی اول $a=2 \to 1=x_1^2+2x_1-2 \to \underline{x_1^2+2x_1-3=0}$ ۸_

$(\underbrace{x_1+3}_{-3})(\underbrace{x_1-1}_{1})=0 \qquad$ سهمی دوم $\to a=2 \to 1=-x_2^2-2x_2+b \to \underline{x_2^2+2x_2+(1-b)=0}$

$\Delta_1=2^2-4(-3)=4+12=16 \xrightarrow{\text{عرض}} \frac{\overset{\sqrt{16}}{\sqrt{\Delta}}}{|a|}=4$

$\Delta_2=2^2-4(1-b)=4-4+4b=4b \xrightarrow[\text{عرض}]{} \frac{\sqrt{4b}}{\underbrace{|a|}_{1}}=2\sqrt{b}=4 \Rightarrow \boxed{b=4}$

$a\times b=2\times 4=8$

$2x^2-ax+b=0 \quad A \to \alpha, \beta \qquad \alpha+\beta=(r+s)+1 \;,\; \alpha\cdot\beta=(r+s)\frac{1}{2}+\frac{1}{2}(r+s)+\frac{1}{4}$ ۹_

$=(r+s)+\frac{1}{4}$

$4x^2+ax-9=0 \quad B \to r, s \qquad A: \alpha+\beta=-\frac{-a}{2}=\frac{a}{2} \;,\; \alpha\cdot\beta=\frac{b}{2}$

$B: r+s=\frac{-a}{2a}=\frac{-1}{2} \quad , \quad r\times s=\frac{-6}{2a}=\frac{-3}{a}$

$\alpha+\beta=(r+s)+1=\frac{-1}{2}+1=\frac{1}{2} \qquad \frac{a}{2}=\frac{1}{2} \Rightarrow \boxed{a=1}$

$A: 2x^2-x+b=0 \rightarrow \alpha+\beta=\frac{1}{2} \ , \ \alpha\cdot\beta=\frac{b}{2}$

$B: 2x^2+x-6=0 \Rightarrow s+r=\frac{-1}{2} \ , \ r\cdot s=\frac{-3}{1}=-3$

$\rightarrow \alpha\beta=(r+\frac{1}{2})(s+\frac{1}{2})=rs+\frac{1}{2}(r+s)+\frac{1}{4}=-3-\frac{1}{4}+\frac{1}{4}=-3$

$\alpha\beta=\frac{b}{2}=-3 \Rightarrow \boxed{b=-6} \Rightarrow \left[\frac{ab}{4}\right]=\left[\frac{-6}{4}\right]=[-1.5]=\boxed{-2}$

$\alpha^2+6\alpha+m=0 \qquad \alpha^2+2\alpha-3m=0 \qquad m=-\alpha^2-6\alpha \qquad \alpha^2+2\alpha-3(-\alpha^2-6\alpha)=0$

$\alpha^2+2\alpha+3\alpha^2+18\alpha=0 \rightarrow 4\alpha^2+20\alpha=0 \qquad 4\underbrace{\alpha}_{0}(\underbrace{\alpha+5}_{-5})=0$

$(-5)^2+6(-5)+m=0 \qquad \boxed{m=-5}$

$x^2+6x+5=0 \rightarrow (x+1)(x+5)=0$ -1 غیر مشترک -5 مشترک

$x^2+2x-3(5)=0 \rightarrow x^2+2x-15=0$

$(\underbrace{x+5}_{-5 \text{ م}})(\underbrace{x-3}_{3 \text{ غ م}})=0$

$\Rightarrow |3-(-1)|=3+1=\boxed{4}$